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Chapter-33

D.C. CIRCUITS

Complex Network:

"A circuit which consists of a number of resistors, capacitors and voltage sources is called a complex network"

Branch:

"Each resistance in the circuit is called a branch."

Node:

"The point where two or more braches meet is called a node or junction."

Datum Node:

"The junction of maximum number of branches is called datum node. It is also called reference node."

Loop or Mesh:

If a number of branches in a network form a closed path. They are said to form a loop or mesh.

A complex network can be solved by using Kirchoff's rules.

Kirchhoff's Ist Rule:

According to this rule;

The sum of all the currents flowing towards a node is equal to the sum of all the currents flowing away from the node."

Mathematically this law is expressed as

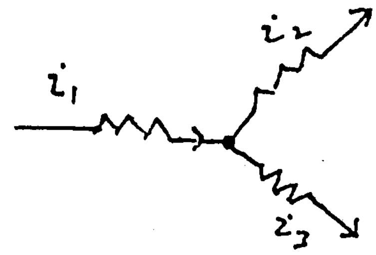
$$\sum_{j=1}^{j=n} i_j = 0.$$

The current flowing towards a node are taken as +ve and the currents flowing away from the node are taken as -ve.

Fig. shows a node N of three branches.

The currents i_1 is flowing towards N and currents i_2 and i_3 are flowing away from N.

Then according to this rule



$$i_1 = i_2 + i_3$$

OR $$i_1 - i_2 - i_3 = 0$$

Kirchhoff's first rule is also called junction rule or junction theorem. It is simply a statement of law of conservation of charge according to which charge is conserved at all points of the circuit.

"According to this rule the algebraic sum of voltage changes around a closed circuit is zero."

This rule is also called loop rule or loop theorem. It is simply a statement of law of conservation of energy.

Mathematically this law is expressed as

$$\sum E_j = 0.$$

The above two laws are called circuit theorems.

1- Calculating the Current in a Single Loop: (On the basis of Law of Conservation of Energy)

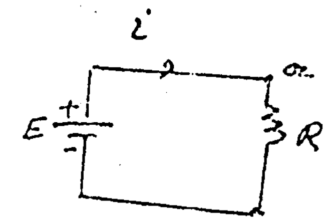
Consider a single loop circuit consisting of a source of emf 'E' and a resistance R.

Let dq amount of charge passes through the circuit in time dt.

Then current in the loop is

$$i = \frac{dq}{dt}$$

$$\therefore dq = i\,dt.$$



The energy dissipated in resistor R is in the form of internal energy as,

$$du = i^2 R\,dt.$$

During this time dt, battery does work dw in sending charge dq from -ve to +ve terminal.

$$dw = E i\,dt.$$

But by law of conservation of energy, the internal energy and work dw are same.

$$\therefore i^2 R\,dt = E i\,dt.$$

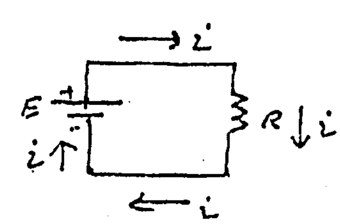
$$E = iR.$$

$$\boxed{i = E/R}$$

This is the expression for current in a single loop circuit on the basis of law of conservation of energy.

(b) On the basis of Potential Concept:

In fig. (a) if we start from point (a) at potential 'V_a'. We go around the circuit clockwise. In going through the resistor, there is a change in potential = $-iR$. where -ve sign shows that top of resistor is having higher potential than the bottom. This is true because +ve charge moves from higher to lower potential.



When we traverse the battery from bottom to top; the potential increases to +E because battery has to do work in moving the +ve charge from low potential to high potential.

Adding the algebraic sum of charges in potential to the initial potential V_a, we must reach the same final value V_a. i.e

$$\cancel{V_a} - iR + E = V_a.$$

$$-iR + E = 0$$

or $$E = iR$$

$$\boxed{i = \frac{E}{R}}$$

This is the expression for current based on potential concept.. Hence two methods for calculating the current in a single loop give similar results.

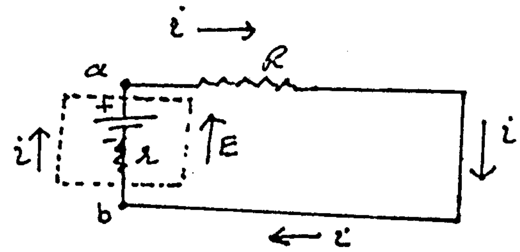
Rules for finding pot-difference

(i) If the resistor 'R' is traversed in the direction of current 'i', the change in potential is $-iR$ and in opposite direction it is $+iR$.

(ii) The emf of a battery is taken as +ve if battery is traversed from -ve to +ve and vice versa.

Internal Resistance of a Cell:

This equation $i = \frac{E}{R}$ is obtained by assuming that internal resistance of the battery is zero.. But internal resistance can't be ignored.



If r is the internal resistance of the battery and we start from point b and go around the circuit clockwise then by loop rule we get,

$$V_b + E - ir - iR = V_b.$$

$$E - ir - iR = 0.$$

$$E = ir + iR.$$

$$E = i(r+R)$$

$$\boxed{i = \frac{E}{r+R}}$$

In writing this equation we have traversed r and R in the direction of current and we have traversed the battery from -ve to +ve.

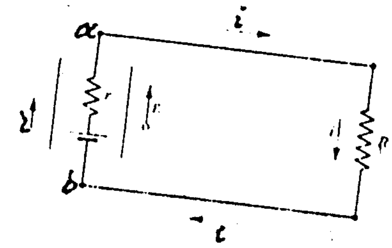
It should be noted that the same equation will be obtained if we go around the circuit anti-clock wise.

From the above equation we see that current decreases due to internal

2- Voltage at Various Elements of a Loop:

Let us find the potential difference b/w points 'a' and 'b' of the fig. shown.

Suppose we start from potential 'b' & traverse the circuit anticlockwise to potential (a).



If V_a and V_b are the potentials at points 'a' and 'b', then

$$V_b + iR = V_a.$$

Here iR is +ve because we are traversing R opposite to the direction of current. When we move opposite to the direction of current & the potential increases

$$\therefore V_a > V_b$$

$$V_a - V_b = iR.$$

$$\boxed{\therefore V_a - V_b = \frac{ER}{R+r}} \quad — (1) \qquad \because i = \frac{E}{R+r}$$

If we start from point (a) & traverse the circuit anticlockwise through the source of emf then

$$V_a + ir - E = V_b.$$

$$\text{or} \quad V_a - V_b = E - ir.$$

$$\therefore V_a - V_b = E - \frac{E}{R+r} r \qquad \because i = \frac{E}{R+r}$$

$$= E\left(1 - \frac{r}{R+r}\right).$$

$$= E\left(\frac{R+r-r}{R+r}\right)$$

$$\boxed{V_a - V_b = \frac{ER}{R+r}} \quad — (2)$$

In (1) and (2) we find that potential difference is same in both

the case. So potential difference is independent of path.
In equations ① and ② the quantity $V_a - V_b$ is the potential difference across the battery terminals.
From equations ① and ② we also see that

$$V_a - V_b = E.$$

If battery has zero internal resistance i.e $r = 0$ or the circuit is open. i.e $R = \infty$.

Sample Problem-1

Sample Problem 1 What is the current in the circuit of Fig. 4a? The emfs and the resistors have the following values:

$\mathcal{E}_1 = 2.1$ V, $\mathcal{E}_2 = 4.4$ V,

$r_1 = 1.8\ \Omega$, $r_2 = 2.3\ \Omega$, $R = 5.5\ \Omega$.

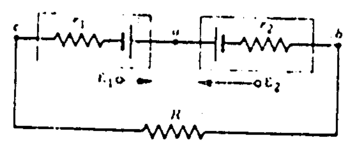


Fig 4a

Sol. $E_1 = 2.1$ Volt ; $E_2 = 4.4$ Volt.

$r_1 = 1.8\,\Omega$; $r_2 = 2.3\,\Omega$, $R = 5.5\,\Omega$.

By loop rule starting from point (a) in the clockwise direction, we have,

$$-E_2 + ir_2 + iR + ir_1 + E_1 = 0.$$

$$\therefore ir_1 + ir_2 + iR = E_2 - E_1.$$

$$i(r_1 + r_2 + R) = E_2 - E_1.$$

$$i = \frac{E_2 - E_1}{r_1 + r_2 + R}$$

Putting the values we get,

$$i = \frac{4.4 - 2.1}{1.8 + 2.3 + 5.5} = \frac{2.3}{9.6} = 0.239.$$

$$\boxed{i = 0.24 \text{ Ampere}} \quad \text{Ans}$$

Sample Problem-2

(a) What is the potential difference between points a and b in Fig. 4a? (b) What is the potential difference between points a and c in Fig. 4a?

See above. in S.P 1

Fig 4a

Sol $E_1 = 2.1$V, $E_2 = 4.4$V. $r_1 = 1.8\,\Omega$

$r_2 = 2.3\,\Omega$, $R = 5.5\,\Omega$, $i = 0.24$ Amp.

(a) $V_{ab} = ?$. (b) $V_{ac} = ?$.

(a) Starting from potential 'b' in anticlock... On we have

$$V_b - i r_2 + E_2 = V_a.$$

or $V_a - V_b = E_2 - i r_2.$

$= 4.4 - 0.24 \times$ ... $.4 - 0.55$

$$\boxed{\therefore V_{ab} = 3.8 \text{ volts}}.$$

(b) Starting from potential 'c' going c : ku . , we have

$$V_c + i r_1 + E_1 = V_a.$$

or $V_a - V_c = E_1 + i R_1.$

$= 2.1 + 0.24 \times 1.8 =$ ... 4

$$\boxed{\therefore V_{ac} = 2.5 \text{ Volt}}$$ Ans

3. Multiloop Circuits:

The circuit having more than one loop are called multiloop circuits.

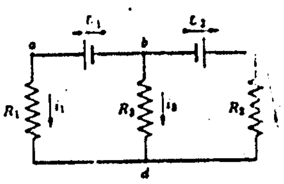


Fig. shows a complex net work consisting of three resistors. R_1, R_2 and R_3 and two batteries E_1 and E_2. For simplicity we neglect the internal resistances of the batteries.

There are two nodes in the circuit 'b and 'd'. In a multiloop circuit the node is a point at which three or more wire segments meet. In ... fig. shown points b and d are nodes. Points 'a' and 'c' are not n... s because only two wire segments meet

In a single loop circuit there is only one current. But in a multilo... circuit there are more than one currents to determine.

In the circuit shown there are three currents i_1, i_2 and i_3. i_1 is the current flowing through R_1, i_2 is the current flowing through R_2 and i_3 is the current flowing through R_3. The point d is taken as the datum node.

Current i_1 and i_3 are flowing towards 'd' while i_2 is flowing away from 'd'. So we can write

$$\boxed{i_2 + i_3 = i_2} \quad \text{—— ①}$$

This equation ① represents Kirchoffs 1ste rule known as current rule according to which,

"The sum of currents flowing Towards the node is equal to the sum of currents flowing away from the node."

Using Kirchoff's IInd rule in anticlockwise direction to Ist loop by starting ending at point (b) we get

$$\boxed{E_1 - i_1 R_1 + i_3 R_3 = 0} \quad \text{— ②}$$

Using Kirchhoff's IInd rule to loop IInd in anticlockwise direction. starting and ending at b, we get

$$\boxed{-i_3 R_3 - i_2 R_2 - E_2 = 0} \quad \text{— ③}$$

Now find i_1, i_2 and i_3, we solve equations ①, ② and ③ simultaneous as follows

from eq. ②

$$i_3 R_3 = i_1 R_1 - E_1.$$

$$\boxed{\therefore \quad i_3 = \left(\frac{i_1 R_1 - E_1}{R_3}\right)} \quad \text{—— (A).}$$

Putting i_3 in ③ we get

$$-\left(\frac{i_1 R_1 - E_1}{R_3}\right) R_3 - i_2 R_2 - E_2 = 0.$$

$$-i_1 R_1 + E_1 - i_2 R_2 - E_2 = 0.$$

$$E_1 - E_2 - i_1 R_1 = i_2 R_2.$$

$$\boxed{i_2 = \frac{E_1 - E_2 - i_1 R_1}{R_2}} \quad \text{— (B)}$$

Putting the values of i_3 and i_2 from (A) and (B) in ① we get

$$i_1 + i_3 = i_2 \quad \text{— ①}$$

$$i_1 = i_2 - i_3.$$

$$\therefore \quad i_1 = \left(\frac{E_1 - E_2 - i_1 R_1}{R_2}\right) - \left(\frac{i_1 R_1 - E_1}{R_3}\right)$$

$$\therefore \quad i_1 = \frac{R_3(E_1 - E_2 - i_1 R_1) - R_2(i_1 R_1 - E_1)}{R_2 R_3}$$

$$i_1 R_2 R_3 = E_1 R_3 - E_2 R_3 - i_1 R_1 R_3 - i_1 R_1 R_2 + E_1 R_2 .$$

$$i_1 R_2 R_3 + i_1 R_1 R_3 + i_1 R_1 R_2 = E_1 R_3 - E_2 R_3 + E_1 R_2$$

$$i_1 (R_2 R_3 + R_1 R_3 + R_1 R_2) = E_1 R_3 + E_1 R_2 - E_2 R_3 .$$

$$i_1 (R_1 R_3 + R_2 R_3 + R_1 R_2) = E_1 (R_2 + R_3) - E_2 R_3 .$$

$$\boxed{i_1 = \frac{E_1 (R_2 + R_3) - E_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}} \quad \text{— (i)}$$

Now adding (2) and (3) we get

$$E_1 - i_1 R_1 - i_2 R_2 - E_2 = 0$$

$$i_1 R_1 = E_1 - E_2 - i_2 R_2 .$$

$$\boxed{i_1 = \frac{E_1 - E_2 - i_2 R_2}{R_1}} \quad \text{— (a)}$$

From eq. (3)

$$i_3 R_3 = -i_2 R_2 - E_2$$

$$\boxed{i_3 = \frac{-i_2 R_2 - E_2}{R_3}} \quad \text{— (b)} .$$

Putting (i_1) and (i_3) from (a) and (b) in (1) we get

$$i_1 + i_3 = i_2 \quad \text{— (1)}$$

$$\frac{E_1 - E_2 - i_2 R_2}{R_1} + \frac{-i_2 R_2 - E_2}{R_3} = i_2$$

$$i_2 = \frac{E_1 R_3 - E_2 R_3 - i_2 R_2 R_3 - i_2 R_1 R_2 - E_2 R_1}{R_1 R_3}$$

$$= \frac{E_1 R_3 - E_2 R_3 - E_2 R_1 - i_2 R_1 R_2 - i_2 R_2 R_3}{R_1 R_3}$$

$$i_2 R_1 R_3 = E_1 R_3 - E_2 R_3 - E_2 R_1 - i_2 R_1 R_2 - i_2 R_2 R_3$$

$$i_2 R_1 R_3 + i_2 R_1 R_2 + i_2 R_2 R_3 = E_1 R_3 - E_2 R_3 - E_2 R_1 .$$

$$i_2(R_1R_3 + R_1R_2 + R_2R_3) = E_1R_3 - E_2(R_3 + R_1)$$

$$\boxed{i_2 = \frac{E_1R_3 - E_2(R_1 + R_3)}{R_1R_2 + R_2R_3 + R_1R_3}} \quad -(a_2)$$

Now putting the values of i and i_2 from (a_1) and (a_2) in ① we get,

$$i_1 + i_3 = i_2 \quad - ①$$

$$i_3 = i_2 - i_1.$$

$$i_3 = \left[\frac{E_1R_3 - E_2(R_1 + R_3)}{R_1R_2 + R_2R_3 + R_1R_3}\right] - \left[\frac{E_1(R_2 + R_3) - E_2R_3}{R_1R_2 + R_2R_3 + R_1R_3}\right]$$

$$= \frac{E_1R_3 - E_2R_1 - E_2R_3 - E_1R_2 - E_1R_3 + E_2 \dots}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$i_3 = \frac{-E_2R_1 - E_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$\boxed{i_3 = \frac{-E_1R_2 - E_2R_1}{R_1R_2 + R_2R_3 + R_1R_3}} \quad -(a_3)$$

m eq. (a_3) we find that i_3 is always $-$ve whatever be th numerical ues of Emfs and resistances may be. It means that direc n of i_3 is ward. i.e opposite to the direction shown in fig.

uations (a_1) and (a_2) shows that i_1 and i_2 may be $+$ve or $-$ve pending on numerical values of emfs and resistances.

Sample Problem-5

Sample Problem 5 Figure 10 shows a circuit whose elements have the following values:

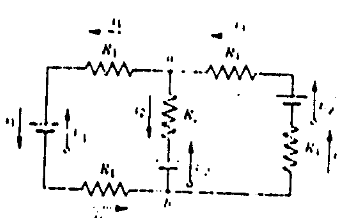
$\mathcal{E}_1 = 2.1$ V, $\mathcal{E}_2 = 6.3$ V,

$R_1 = 1.7\ \Omega$, $R_2 = 3.5\ \Omega$

Find the currents in the three branches of the circuit

$E_1 = 2.1V$, $E_2 = 6.3V$, $R_1 = 1.7\Omega$

$R_2 = 3.5\Omega$, $i_1 = ?$, $i_2 = ?$, $i_3 = ?$



sing Kirchiff's 1st rule at (a) we get

$$i_1 + i_2 = i_3 \quad - ①$$

$$i_2 (R_1R_3 + R_1R_2 + R_2R_3) = E_1R_3 - E_2(R_3+R_1)$$

$$\boxed{i_2 = \frac{E_1R_3 - E_2(R_1+R_3)}{R_1R_2 + R_2R_3 + R_1R_3}} \quad -(a$$

Now putting the values of i and i_2 from (a_1) an ... in ① we get,

$$i_1 + i_3 = i_2 \quad - ①$$

$$i_3 = i_2 - i_1.$$

$$i_3 = \left[\frac{E_1R_3 - E_2(R_1+R_3)}{R_1R_2 + R_2R_3 + R_1R_3}\right] - \left[\frac{E_1(R_2+R_3)}{R_1R_2 + R_2} \quad \frac{E_2R_3}{+R_1R_3}\right]$$

$$= \frac{E_1R_3 - E_2R_1 - E_2R_3 - E_1R_2 - E_1R_3 + E \ldots R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$i_3 = \frac{-E_2R_1 - E_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$\boxed{i_3 = \frac{-E_1R_2 - E_2R_1}{R_1R_2 + R_2R_3 + R_1R_3}} \quad -(a$$

From eq.(a_3) we find that i_3 is always $-$ve whatever be the numerical values of Emfs and resistance may be. It means that ... direction of i_3 is upward. i.e opposite to the direction shown in fig.

Equations (a_1) and (a_2) shows that i_1 and i_2 may be +ve or -ve depending upon numerical values of emfs and resistances.

Sample Problem-5

Sample Problem 5 Figure 10 shows a circuit whose ... ents have the following values:

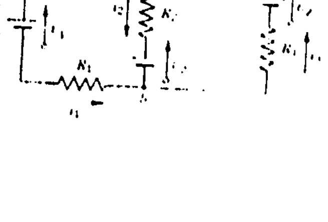
$\mathcal{E}_1 = 2.1$ V, $\mathcal{E}_2 = 6.3$ V,
$R_1 = 1.7\,\Omega$, $R_2 = 3.5\,\Omega$.

Find the currents in the three branches of the circuit

Sol: $E_1 = 2.1V$, $E_2 = 6.3V$, $R_1 = 1.7\Omega$

$R_2 = 3.5\Omega$, $i_1 = ?$, $i_2 = ?$, $i_3 = ?$

Using Kirchoff's 1st rule at (a) we get



$$i_1 + i_2 = i_3 \quad - ①$$

Using Kirchhoff's IInd rule on left loop in anticlockwise direction and starting from point (a) we get

$$-i_1 R_1 - E_1 - i_1 R_1 + E_2 + i_2 R_2 = 0.$$

$$\text{or} \quad -2i_1 R_1 - E_1 + E_2 + i_2 R_2 = 0.$$

$$E_2 - E_1 = 2i_1 R_1 - i_2 R_2 \quad —② $$

Using Kirchhoff's IInd rule on right loop in anticlockwise direction starting from point (a) we get.

$$-i_2 R_2 - E_2 - i_3 R_1 + E_2 - i_3 R_1 = 0.$$

$$-i_2 R_2 - 2i_3 R_1 = 0.$$

$$\text{or} \quad i_2 R_2 + 2i_3 R_1 = 0 \quad — ③$$

Now we solve ①, ② and ③ as follows. From eq. ③

$$\boxed{i_3 = \frac{-i_2 R_2}{2R_1}}$$

Putting the value of i_3 in eq. ① we get,

$$i_1 + i_2 = i_3 \quad — ①$$

$$i_1 + i_2 = \frac{-i_2 R_2}{2R_1}$$

$$i_1 = \frac{-i_2 R_2}{2R_1} - i_2$$

$$i_1 = i_2 \left(\frac{-R_2}{2R_1} - 1 \right)$$

$$i_1 = i_2 \left(\frac{-R_2 - 2R_1}{2R_1} \right) \quad — (A)$$

From eq. ②

$$i_2 R_2 = E_1 - E_2 + 2i_1 R_1$$

$$i_2 = \frac{E_1 - E_2 + 2i_1 R_1}{R_2}$$

Putting this value of i_2 in eq. (A) we get

$$i_1 = \left[\frac{E_1 - E_2 + 2i_1 R_1}{R_2} \right] \left[\frac{-R_2 - 2R_1}{2R_1} \right]$$

$$i_1 = \left(\frac{E_2 - E_1 - 2i_1 R_1}{R_2} \right) \left(\frac{R_2 + 2R_1}{2R_1} \right)$$

$$i_1 = \frac{(E_2 - E_1)(R_2 + 2R_1) - 2i_1 R_1 (R_2 + 2R_1)}{2R_2 R_1}$$

$$2R_1R_2i_1 = (E_2-E_1)(R_2+2R_1) - 2i_1R_1(R_2+2R_1).$$

$$2R_1R_2i_1 + 2R_1i_1(R_2+2R_1) = (E_2-E_1)(R_2+2R_1).$$

$$i_1[2R_1R_2 + 2R_1(R_2+2R_1)] = (E_2-E_1)(R_2+2R_1).$$

$$i_1 2R_1[R_2 + (R_2+2R_1)] = (E_2-E_1)(R_2+2R_1).$$

$$2i_1R_1(2R_2+2R_1) = (E_2-E_1)(R_2+2R_1).$$

$$4i_1R_1(R_1+R_2) = (E_2-E_1)(R_2+2R_1).$$

$$\boxed{i_1 = \frac{(E_2-E_1)(R_2+2R_1)}{4R_1(R_1+R_2)}} \quad - (A')$$

Putting the values of E_1, E_2, R_1 and R_2 we get

$$i_1 = \frac{(6.3-2.1)(3.5+2\times1.7)}{4\times1.7(1.7+3.5)}$$

$$= \frac{4.2\times6.9}{6.8(5.2)} = \frac{28.98}{35.36} = 0.819.$$

$$\boxed{i_1 = 0.82\,A} \quad \text{Ans.}$$

Putting i_1 from (A) in equation (2) we get

$$(E_2-E_1) = 2i_1R_1 - i_2R_2 \quad - (2)$$

$$\therefore \quad i_2R_2 = 2i_1R_1 - (E_2-E_1)$$

$$i_2R_2 = 2\left[\frac{(E_2-E_1)(R_2+2R_1)}{4R_1(R_1+R_2)}\right]R_1 - (E_2-E_1)$$

$$i_2R_2 = \frac{(E_2-E_1)(R_2+2R_1)}{2(R_1+R_2)} - (E_2-E_1)$$

$$= (E_2-E_1)\left[\frac{R_2+2R_1}{2(R_1+R_2)} - 1\right]$$

$$i_2R_2 = \frac{(E_2-E_1)[R_2+2R_1-2(R_1+R_2)]}{2(R_1+R_2)}$$

$$i_2R_2 = \frac{(E_2-E_1)(-R_2)}{2(R_1+R_2)}$$

$$\boxed{i_2 = \frac{-(E_2-E_1)}{2(R_1+R_2)}} \quad - (B).$$

$$2R_1R_2 i_1 = (E_2 - E_1)(R_2 + 2R_1) - 2i_1R_1(R_2 + 2R_1).$$

$$2R_1R_2 i_1 + 2R_1 i_1 (R_2 + 2R_1) = (E_2 - E_1)(R_2 + 2R_1).$$

$$i_1 [2R_1R_2 + 2R_1(R_2 + 2R_1)] = (E_2 - E_1)(R_2 + 2R_1).$$

$$i_1 2R_1 [R_2 + (R_2 + 2R_1)] = (E_2 - E_1)(R_2 + 2R_1).$$

$$2i_1R_1 (2R_2 + 2R_1) = (E_2 - E_1)(R_2 + 2R_1).$$

$$4i_1R_1 (R_1 + R_2) = (E_2 - E_1)(R_2 + 2R_1).$$

$$\boxed{i_1 = \frac{(E_2 - E_1)(R_2 + 2R_1)}{4R_1(R_1 + R_2)}} \quad \text{— (A)}$$

Putting the values of E_1, E_2, R_1 and R_2 we get

$$i_1 = \frac{(6.3 - 2.1)(3.5 + 2 \times 1.7)}{4 \times 1.7(1.7 + 3.5)}$$

$$= \frac{4.2 \times 6.9}{6.8(5.2)} = \frac{28.98}{35.36} = 0.819.$$

$$\boxed{i_1 = 0.82\ A} \quad \text{Ans.}$$

Putting i_1 from (A) in equation (2) we get

$$(E_2 - E_1) = 2i_1R_1 - i_2R_2 \quad \text{— (2)}$$

$$\therefore \quad i_2R_2 = 2i_1R_1 - (E_2 - E_1)$$

$$i_2R_2 = 2\left[\frac{(E_2 - E_1)(R_2 + 2R_1)}{4R_1(R_1 + R_2)}\right]R_1 - (E_2 - E_1)$$

$$i_2R_2 = \frac{(E_2 - E_1)(R_2 + 2R_1)}{2(R_1 + R_2)} - (E_2 - E_1)$$

$$= (E_2 - E_1)\left[\frac{R_2 + 2R_1}{2(R_1 + R_2)} - 1\right]$$

$$i_2R_2 = \frac{(E_2 - E_1)[R_2 + 2R_1 - 2(R_1 + R_2)]}{2(R_1 + R_2)}$$

$$i_2R_2 = \frac{(E_2 - E_1)(-R_2)}{2(R_1 + R_2)}$$

$$\boxed{i_2 = \frac{-(E_2 - E_1)}{2(R_1 + R_2)}} \quad \text{— (B).}$$

Putting the values of E_1, E_2, R_1 and R_2 we get.

$$i_2 = \frac{-(6.3 - 2.1)}{2(1.7+3.5)} = \frac{-4.2}{10.4}$$

$$i_2 = -0.40 \text{ Amp.} \quad \text{Ans.}$$

The -ve sign shows that direction of i_2 is opposite to the direction shown in fig

Now putting (A') and (B) in ① we get

$$i_1 + i_2 = i_3 \quad — ①.$$

$$i_3 = \frac{(E_2 - E_1)(R_2 + 2R_1)}{4R_1(R_1 + R_2)} + \frac{-(E_2 - E_1)}{2(R_1 + R_2)}$$

$$= \frac{(E_2 - E_1)(R_2 + 2R_1) - 2R_1(E_2 - E_1)}{4R_1(R_1 + R_2)}$$

$$\therefore \quad i_3 \quad \frac{(E_2 - E_1)}{4R_1(R_1 + R_2)}\{R_2 + 2R_1 - 2R_1\}$$

$$\therefore \quad = \frac{(E_2 - E_1) R_2}{4R_1(R_1 + R_2)}$$

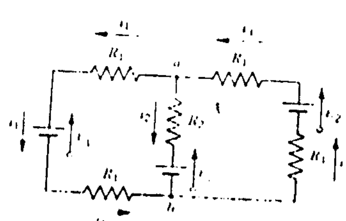
Now putting values of E_1, E_2, R_1 and R_2 we get

$$i_3 = \frac{(6.3 - 2.1) \times 3.5}{4 \times 1.7(1.7+3.5)} = \frac{14.7}{35.36} = 0.415$$

$$\boxed{i_3 = 0.42} \quad \text{Ans.}$$

Sample Problem 6

Sample Problem 6 What is the potential difference between points a and b in the circuit of Fig.



Sol. $E_1 = 2.1V$ $E_2 = 6.3V$, $R_1 = 1.7\Omega$

$R_2 = 3.5\Omega$, $V_{ab} = ?$; ($i_2 = -0.40$ Amp)
(from Sample Prob. 5)

Let us traverse branch R_2 from point a to b.

$$\because V_a - i_2 R_2 - E_2 = V_b.$$

$$\therefore V_a - V_b = i_2 R_2 + E_2.$$

Putting values of i_2, R_2 and E_2 we get.

$\therefore \; V_a - V_b = -0.4 \times 3.5 + 6.3$

$\doteq -1.4 + 6.3$

$V_{ab} = 4.9$ Volt Ans.

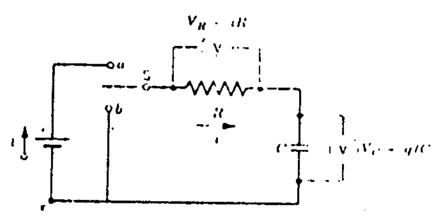
Growth of Charge in an R.C Circuit:--

R.C Circuit

"An R.C circuit consists of a resistance R and capacitor 'C' connected in Series with a battery of emf E."

Fig. shows an R.C circuit in series with a battery of voltage E. For charging the capacitor, switch S is closed at (a) and opened at 'b'.

By using Kirchhoff's II rule and starting from point x clockwise we get



$E - V_R - V_c = 0.$

$E = V_R + V_c.$

Sin R and C are in series

Now $V_R = iR.$

and $V_c = \frac{q}{c}.$

$\therefore \; E = iR + \frac{q}{c}$ — ①.

Where q and i denote the charge and current at any time 't'.

Now $i = \frac{dq}{dt}.$

$\therefore$ Equation ① becomes

$\therefore \; E = \frac{dq}{dt} R + \frac{1}{C} q.$

$EC = \frac{dq}{dt} RC + q.$

or $RC \frac{dq}{dt} = EC - q.$

$\frac{dq}{EC - q} = \frac{1}{RC} dt$

$$-\frac{dq}{EC-q} = -\frac{1}{RC}\,dt.$$

Integration gives

$$\ln(EC-q) = -\frac{1}{RC}t + A \quad \text{— ②}$$

Where A is constant of integration. To find 'A' we use initial condition.

i.e at $t=0$; $q=0$ $\therefore \ln EC = A$.

$\therefore$ Eq. ② becomes

$$\ln(EC-q) = -\frac{1}{RC}t + \ln EC.$$

$$\ln(EC-q) - \ln EC = -\frac{1}{RC}t.$$

$$\ln\frac{EC-q}{EC} = -\frac{1}{RC}t.$$

$$\text{or} \quad \frac{EC-q}{EC} = e^{-t/RC}.$$

$$EC-q = EC\,e^{-t/RC}.$$

$$\text{or} \quad q = EC - EC\,e^{-t/RC}$$

$$q = EC(1-e^{-t/RC}) \quad \text{— ③}$$

This equation shows that for $t=\infty$; $q = EC = q_0$. where q_0 is the maximum value of charge on the capacitor.

$$\therefore \quad q = q_0(1-e^{-t/RC}). \quad \text{— (A)}$$

The equation (A) gives the growth of charge in the circuit. The eq. shows that charge 'q' goes on increasing with time and ultimatlly attains max. value q_0 after a long time.

So we find that charge of capacitor increases exponentialy with time.

Capacitive Time Constant (τ_c)

In eq. (A) the factor RC has the dimension of time and is called capacitance time constant. It is denoted by τ_c.

$\therefore$ Equation (A) becomes

$$q = q_0(1-e^{-t/\tau_c}) \quad \text{— (a)}.$$

At $t=0$; $q=0$, But $t=\infty$; $q = q_0 = EC$.

At $t = \tau_c$

$$q = q_0 \left(1 - e^{-t/\tau_c}\right) - q_0 \left(1 - e^{-1}\right).$$

$$= q_0 \left(1 - \frac{1}{e}\right) = q_0 \left(1 - \frac{1}{2.718}\right).$$

$$q = q_0 (1 - 0.37).$$

$$= q_0 (0.63)$$

$$q = 0.63 q_0.$$

$$q = 63 \text{ of } q_0.$$

So capacit time constant is the time after which the charge on the capacitor grows to 0.63 of its maximum value or it is the time after which capacitor has charge equal to 63% of the final (max) value $q_0 = CE$.

So we find that growth of charge depends on τ_c. The smaller the value of τ_c, the more rapid the growth of charge

Fig. (i) shows the growth of charge.

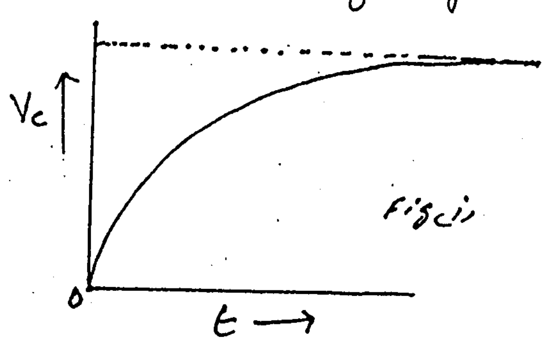
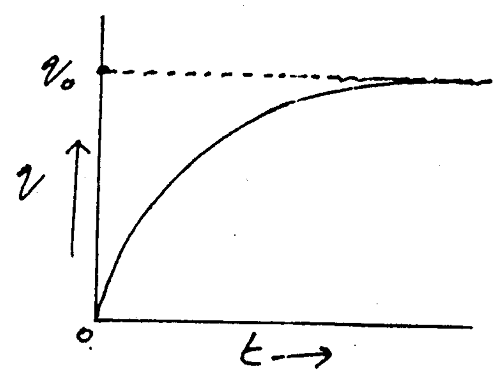


Fig (i)



From the graph we see that in the presence of resistance R, the charging of the capacitor to max. value is delayed due to factor RC. But when RC = 0 i.e resistor is absent then capacitor get fully charged quickly.

When switch S is closed at a, the charge on capacitor is initially zero, so the potential difference across the capacitor $V_c = 0$. At this time $E = iR$ and $i = \frac{E}{R}$.

Due to this current the charge flows to capacitor and potential diff. across the capacitor increases with time.

Because Emf is constant. So by the eq.

$$E = V_R + V_c.$$

when V_c increases, V_R should decrease so that E remains constant.

When V_R decreases, so i decreases. This decrease in current increases the charge on capacitor more slowly. With the passage of time, the current i decreases to zero and pot. diff V_R across the resistor R becomes zero. So the entire potential difference of battery appears across capacitor. So V_c is maximum i.e capacitor gets fully charged to $q_0 = CE$.

Expression for Current Charging the Capacitor:

From equ. (a) $\quad q = q_0 (1 - e^{-\frac{t}{\tau}})$.

Differentiating w.r.t. t.

$$\frac{dq}{dt} = \frac{d}{dt}\left[q_0 (1 - e^{-t/\tau})\right]$$

$$i = q_0 \frac{d}{dt}(1 - e^{-t/\tau_c}) \qquad \because \frac{dq}{dt} = i$$

$$= q_0 \left[e^{-t/\tau_c}\left(-\frac{-1}{\tau_c}\right)\right].$$

$$= \frac{q_0}{\tau_c} e^{-t/\tau_c} \qquad \because \tau_c = RC.$$

$$i = \frac{q_0}{RC} e^{-t/RC} \qquad \because \frac{q_0}{C} = E.$$

$$i = \frac{E}{R} e^{-t/RC} \qquad \because \frac{E}{R} = i_0, \text{ max. current.}$$

$$i = i_0 e^{-\frac{t}{RC}}$$

This shows that current decreases with time exponentially. This is the expression for current charging the capacitor.

Fig (ii) shows the variation of charging current with time.

(i) It show that at $t = 0$ the current is max $= i_0$ and by the eq

$$E = V_R + V_c$$

$E = V_R$ and $V_c = 0$ and the capacitor is not charged.

(ii) At $t = \infty$ the current i drops to zero

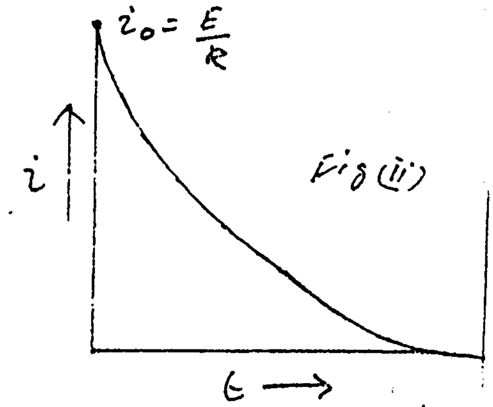


Fig (ii)

and $V_R = 0$ and $E = V_c$ and the capacitor becomes fully charged.

Comparison of Graph (i) and (ii)

A comparison of graph (i) and (ii) shows that when capacitor is fully charged, the charging current is zero or when potential difference across the capacitor increases, the charging current decreases.
So when capacitor becomes fully charged, the current stops flowing.

Sample Problem 7

Sample Problem 7 A resistor R ($= 6.2\ M\Omega$) and a capacitor C ($= 2.4\ \mu F$) are connected in series, and a 12-V battery of negligible internal resistance is connected across their combination. (*a*) What is the capacitive time constant of this circuit? (*b*) At what time after the battery is connected does the potential difference across the capacitor equal 5.6 V?

Sol: $R = 6.2\ M\Omega = 6.2 \times 10^6\ \Omega$. $C = 2.4\ \mu F = 2.4 \times 10^{-6} F$, $E = 12$ Volts.

(a) $\tau_c = RC = ?$

$$\text{As } \tau_c = RC = 6.2 \times 10^6 \times 2.4 \times 10^6$$

$$= 14.88$$

$$\tau_c = 15 . \text{Sec} \quad \text{Ans.}$$

After what time t $V_c = 5.6$ volt.

$$\text{As } q = q_0 (1 - e^{-t/\tau})$$

$\because q_0 = EC$, $\tau_c = RC$

$$\frac{q}{EC} = (1 - e^{-t/RC})$$

$$e^{-t/RC} = 1 - \frac{q}{EC}$$

$$e^{-t/RC} = 1 - \frac{V_c}{E}$$

$$\ln e^{-t/RC} = \ln\left(1 - \frac{V_c}{E}\right)$$

$$-\frac{t}{RC} \ln e = \ln\left(1 - \frac{V_c}{E}\right)$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{V_c}{R}\right)$$

$$-t = RC \ln\left(1 - \frac{V_c}{R}\right)$$

$$t = -RC \ln\left(1 - \frac{V_c}{R}\right)$$

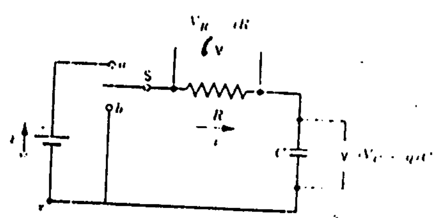
$$t = -\tau_c \ln\left(1 - \frac{V_c}{E}\right)$$

Putting the values of R, V_c, E and τ_c we get.

$$t = -15 \ln\left(1 - \frac{5.6}{12}\right)$$
$$= -15 \ln 0.5333$$
$$= -15 \times -0.628 \quad = 15 \times 0.628$$
$$\boxed{t = 9.4 \text{ sec}} \text{ Ans.}$$

Decay of Charge (Current) in an R.C Series Circuit

Fig. shows an RC circuit in series with a battery of voltage E.
Let us see how capacitor gets discharged.



First close the switch S at (a).
The charge q on the capacitor will go on increasing. Wait for a long time, so that the capacitor gets fully charged and q becomes maximum i.e $q_0 = CE$.
Now when the charge attains maximum value, open the switch S at (a) and close the switch at (b) so the battery is ~~disconnected~~. So $E = 0$.

So $\quad V_R + V_C = 0.$

$\therefore \quad iR + \frac{q}{C} = 0. \qquad$ when $V_R = iR.$

$$V_C = \frac{q}{C}.$$

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad \because i = \frac{dq}{dt}.$$

$$R\frac{dq}{dt} = -\frac{1}{C}q.$$

$$\frac{dq}{q} = -\frac{1}{RC}dt.$$

– By Integrating $\quad \ln q = -\frac{1}{RC}t + A$

To find constt 'A' we use initial conditions. i.e
$q = q_0$ at $t = 0$ - Since initial value of charge was q_0 when battery was removed.

$\therefore \quad A = \ln q_0.$

So above expression becomes,

$$\therefore \ln q = -\frac{1}{RC}t + \ln q_0.$$

$$\ln q - \ln q_0 = -\frac{1}{RC}t.$$

$$\ln \frac{q}{q_0} = -\frac{t}{RC}$$

$$\text{or } \frac{q}{q_0} = e^{-t/RC}$$

$$q = q_0 e^{-t/RC}.$$

$$\therefore q = q_0 e^{-t/\tau_c} \quad \text{—(1).} \qquad \text{But } RC = \tau_c$$

Equation (1) shows that the charge q decreases exponentially with time. This equation shows that for $t = \infty$, $q = 0$ i.e after a long time capacitor gets fully discharged.

Capacitive Time Constt. The product $RC = \tau_c$ is called the capacitive time constant. It appears in the expression for a charging capacitor as well as in the expression for discharging capacitor.

$$\text{As } q = q_0 e^{-t/\tau_c}$$

At $t = 0$, $q = q_0$ At $t = \tau_c$; $q = q_0 e^{-1} = \frac{q_0}{e} = \frac{q_0}{2.718}$.

$$\boxed{q = 0.37 q_0}$$

So after time τ_c the charge of capacitor reduces to 37% of the initial value q_0.

"So the time after which the charge on the capacitor is 0.37 of its maximum value is called capacitive time constt of the circuit."

So we find that decay of charge depends on τ_c, the smaller the value of τ_c the more rapid the decay of charge.

Fig(2) shows variation of charge q with time t.

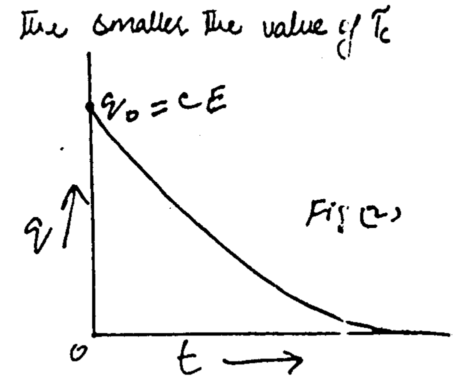


Fig (2)

Discharging Current:

The equation of current for discharging the capacitor is obtained by diff. eq ① w.r.t. 't'.

$$\text{As} \quad q = q_0 e^{-t/\tau_c}.$$

$$\text{As} \quad i = \frac{dq}{dt}.$$

$$\therefore \quad i = \frac{d}{dt} q_0 e^{-t/\tau_c}.$$

$$i = q_0 \frac{d}{dt} e^{-t/\tau_c}$$

$$= q_0 e^{-t/\tau_c}\left(-\frac{1}{\tau_c}\right).$$

$$i = -\frac{q_0}{\tau_c} e^{-t/\tau_c}$$

$$\because \tau_c = RC.$$

$$i = -\frac{q_0}{RC} e^{\frac{-t}{RC}}$$

$$\because \frac{q_0}{C} = E.$$

$$\therefore \quad i = -\frac{E}{R} e^{-t/RC}.$$

$$i = -\frac{E}{R} e^{-t/RC} \quad \text{—②} \qquad R_c = \tau_c.$$

At $t = 0$.

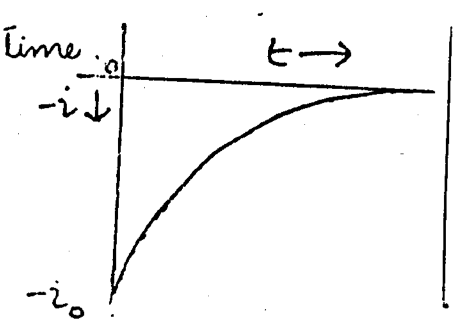
$$i = -\frac{E}{R} e^{-0} = -\frac{E}{R}.$$

But $\frac{E}{R} = i_0$. where i_0 = Max. current.

$$\therefore \quad i = i_0 e^{-t/\tau_c}$$

This is the expression for the current discharging the capacitor. The -ve sign shows that direction of flow of discharging current is opposite to the direction of flow of charging current.

Fig. shows the variation of discharging current with time. It starts with its maximum value and becomes zero after a long time.



Sample Problem - 8

Sample Problem 8 A capacitor C discharges through a resistor R. (a) After how many time constants does its charge fall to one-half its initial value? (b) After how many time constants does the stored energy drop to half its initial value?

Sol (a) after how many time constts does charge fall to one half its initial value?

$$\text{As } q = q_0 e^{-t/\tau_c} \quad \text{— (1).}$$

Let 't' be the time after charge falls to half of initial value.

$\therefore$ Put $q = \frac{q_0}{2}$ in (1). we get.

$$\frac{q_0}{2} = q_0 e^{-t/\tau_c}.$$

$$\frac{1}{2} = e^{-t/\tau_c}.$$

$$2 = e^{t/\tau_c}.$$

$$\ln 2 = \frac{t}{\tau_c} \ln e.$$

$$\therefore 0.693 = \frac{t}{\tau_c} \cdot 1$$

$$t = 0.693\,\tau_c$$

$$\boxed{t = 0.69\,\tau_c} \quad \text{Ans}$$

$\therefore$ The charge drops to half of its initial value after 0.69 time consts.

(b) After how many time constants does the stored energy drop to half of its initial value?

Let 't' be the time after which energy falls to half of its initial value.

As energy of capacitor is given by

$$U = \frac{1}{2} q^2/C$$

$$\therefore U = \frac{1}{2} \frac{q_0^2}{C} e^{-\frac{2t}{\tau_c}} \quad \because q = q_0 e^{-t/\tau_c}. \quad \text{— (1)}.$$

But $\frac{q_0^2}{2C} = U_0$, the initial energy.

$\therefore$ Expression (1) becomes.

$$U = U_0 e^{-\frac{2t}{\tau_c}} \quad \text{— (2)}.$$

But according to given condition

$$U = \frac{U_0}{c}$$

$\therefore$ Equation (2) becomes

$$\frac{U_0}{2} = U_0 e^{-\frac{2t}{\tau_c}}$$

$$\frac{1}{2} = e^{-2t/\tau_c}$$

$$2 = e^{\frac{2t}{\tau_c}}$$

$$\ln 2 = \ln e^{2t/\tau_c}$$

$$\ln 2 = \frac{2t}{\tau_c} \ln e$$

$$\therefore \quad 0.693 = \frac{2t}{\tau_c} \cdot 1$$

$$\frac{0.693\,\tau_c}{2} = t$$

$$t = 0.346\,\tau_c$$

$$\boxed{t = 0.35\,\tau_c} \text{ Ans}$$

$\therefore$ The stored energy drops to half of its initial value after 0.35 time constants.

:- The End :-