Nasir Pervaiz Butt
M.Sc (Physics) M.Phil
Assistant Professor
Govt. College Sargodha.
House No. 333 St. No. 7
Sargodha.

Chapter - 31

CAPACITORS AND DIELECTRICS

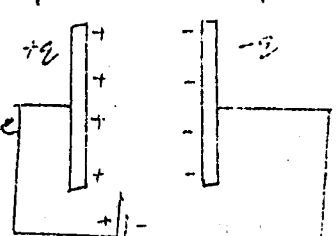
1. Capacitor

"A capacitor is a device used to store charges".

There are many types of capacitors e.g spherical plate capacitor, cylinderical plate capacitor. But the simplest type of capacitor is the parallel plate capacitor. It consists of two parallel metal plates seperated by some insulator called dielectric.

Fig. shows a ||-plate capacitor.

The capacitor is charged by connecting its plate to the opposite terminals of a battery. The plate connected to +ve terminal of the battery acquires +ve charge and the other plate gets -ve charge.



When we say that a capacitor has charge q, it means that one plate has $+q$ charge and the other plate has $-q$ charge. However the net charge on the capacitor is zero.

When a charge is given to a plate, its potential rises. So the magnitude of charge 'q' is directly propotional to the potential difference across the plates. i.e $q \propto V$

$$q = CV.$$

where 'C' is the constant of propotionality called capacitance of the capacitor. Its value depends on the shape of the plates and medium between the plates. It is given by

$$C = q/V$$

It is defined as "The ability of a capacitor to store charge." Or

It is the ratio charge to the resulting potential of the plate.
The S.I unit of capacitance is Farad.

SI is given as

$$1 \text{ Farad} = \frac{1 \text{ coulomb}}{1 \text{ Volt}}$$

"The capacitance is said to be one Farad if a charge of one coulomb is given to one plate produces a potential difference of one volt b/w them".

Farad is a bigger unit, the smaller units are

(i) Micro Farad $= \mu F = 10^{-6} F$.

(ii) Micro Micro Farad $= \mu\mu F = 10^{-12} F$.

(iii) Nano Farad $= nF = 10^{-9} F$. (iv) Femto Farad $= fF = 10^{-15} F$.

$\mu\mu F$ is also called Pico Farad.

Sample Problem - 1

Sample Problem 1 A storage capacitor on a random access memory (RAM) chip has a capacitance of 55 fF. If it is charged to 5.3 V, how many excess electrons are there on its negative plate?

Solution:- $C = 55 fF = 55 \times 10^{-15} F$.

$V = 5.3 \text{ Volt}$; $e = 1.6 \times 10^{-19} C$.

Number of excess electrons on -ve plate = ?

Let N be the number of excess electrons on -ve plate each of charge e. Then total charge is $q = Ne$.

$$N = \frac{q}{e} \qquad \because q = CV.$$

$$= \frac{CV}{e}$$

$$= \frac{55 \times 10^{-15} \times 5.3}{1.6 \times 10^{-19}} = \frac{55 \times 5.3}{1.6} \times 10^{-15+19}$$

$$= 182 \times 10^{4}.$$

$$\boxed{N = 1.8 \times 10^{6} \text{ electrons}} \text{ Ans}$$

2. Calculation of Capacitance:

"Ability of a capacitor to store charge is called its capacitance."

To calculate the capacitance, we should keep in mind the following steps.

(i) We suppose that charge on the plates of capacitor is 'q'.

(ii) We calculate the electric field $\vec{E}$ b/w the plates in terms of charge q by using Gauss's law.

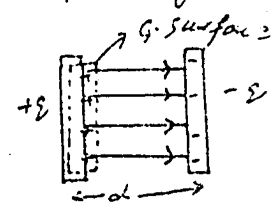
(iii) After knowing $\vec{E}$, we calculate the pot. diff V b/w the plates.

(iv) We calculate capacitance C by the relation $C = \frac{q}{V}$.

Now we study the calculation of $\vec{E}$ and V one by one.

(a) Calculation of Electric Field in a Capacitor

Consider a parallel plate capacitor having each plate of area A. Then by Gauss's law, the flux is given by,



$$\oint \vec{E} \cdot \vec{dA} = \frac{1}{\epsilon_0} \times q.$$

$$\epsilon_0 \oint \vec{E} \cdot \vec{dA} = q \quad \text{—} \ ①.$$

Where q is the charge with in the Gaussian surface and the integration is carried out over the Gaussian surface.

Suppose the length of plate is very large as compared to the distance b/w them. So $\vec{E}$ is uniform and there is no fringing field at the ends. So $\vec{E}$ has constant magnitude. and $\vec{E}$ and $\vec{dA}$ are parallel.

$$\therefore \ \epsilon_0 \oint \vec{E} \cdot \vec{dA} = \oint E \, dA \cos 0^\circ$$

$$\epsilon_0 E \oint dA = \epsilon_0 E A \text{. where A is area of}$$

G. surface through which flux is passing.

$\therefore$ Eq. ① becomes

$$\epsilon_0 E A = q.$$

$$\text{or} \quad \boxed{E = \frac{q}{\epsilon_0 A}}$$

∴ This is the expression for electric field of a capacitor.

(b) Calculation of a Potential Difference:

The potential difference V and the electric field $\vec{E}$ are related as,

$$V_f - V_i = -\int_i^f \vec{E}\cdot\vec{ds}.$$

Where integral is taken over the path starting from +ve plate and ending at -ve plate. For this path, E and $\vec{ds}$ are in the same direction.

∴ The above equation becomes

$$V_f - V_i = -\int_i^f E\,ds$$

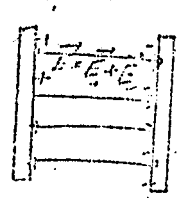
$$V = -\int_i^f E\,ds.$$

The absolute value of pot. diff b/w the plates is given by,

$$V = \int_-^+ E\,ds \quad \text{—} ①$$

where +ve and -ve sign show that path of integration starts from +ve plate and ends at -ve plate.

Now the electric field b/w the plates of capacitor is the sum of the fields due to two plates.



i.e. $\vec{E} = \vec{E}_+ + \vec{E}_-$ where $\vec{E}_+$ is the field due to +ve plate and $\vec{E}_-$ is the field due to -ve plate.

By Gauss's law $\vec{E}_+$ and $\vec{E}_-$ both are proportional to q. So $\vec{E}$ is also proportional to q. So from eq. ① V is also proportional to q i.e. if q is doubled then $\vec{E}$ and V are also doubled. So ratio of q and V i.e. $\frac{q}{V}$ remains constant. This ratio q/V is called capacitance. Now knowing V and q, we can calculate the capacitance of different types of capacitors.

3- Capacitance of a parallel plate capacitor:

Consider a parallel plate capacitor.

The size of the plates is very large and distance b/w the plates is very

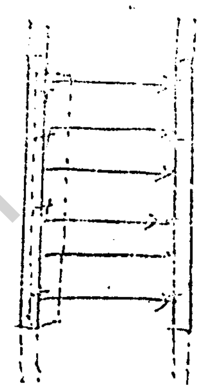
small. So the field is uniform and there is no fringing field at the ends. So $\vec{E}$ is constant throughout the volume b/w the plates.
We draw a Gaussian surface that includes the charge q on the plate.

Now electric field is given by the relation as,

$$E = \frac{q}{\epsilon_0 A} \quad \text{—(i)}$$ where A is the area of each plate.

The potential difference b/w the plates is given by the relation as:-

$$V = \int_{+}^{-} E\, ds.$$



Putting E from (i) we get,

$$V = \frac{q}{\epsilon_0 A} \int_{+}^{-} ds$$

$$= \frac{q}{\epsilon_0 A} \int_{0}^{d} ds$$

$$V = \frac{q}{\epsilon_0 A} d.$$

But $V = q/C$

$$\therefore \frac{q}{C} = \frac{q}{\epsilon_0 A} d.$$

$$\frac{1}{C} = \frac{d}{\epsilon_0 A}.$$

$$\boxed{\therefore C = \frac{\epsilon_0 A}{d}}$$

This is the expression for capacitance of a parallel plate capacitor with free space as the medium b/w the plates.
From the expression we find that C depends on the medium b/w the plates and geometry (Area, distance b/w plates) of the capacitor.
From the expression of capacitance we find that unit of permittivity of free space is also written as

$$\epsilon_0 = \frac{Cd}{A}.$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{Fm}{m^2}.$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m.$$

$$\boxed{\epsilon_0 = 8.85\ pF/m}$$

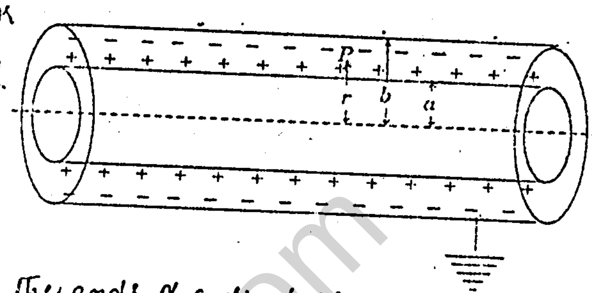
As other unit of ϵ_0 is

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$.

It can be written as $C^2/N\text{-}m^2 = F/m$.

4- Capacitance of a Cylinderical Capacitor:

Consider a cylinderical capacitor of length L formed by two co-axial cylinders of radii 'a' and 'b'. We suppose that L >> b. So that there is no fringing field at the ends of cylinders.



We take a Gaussian surface in the form of a cylinder of length L and radius 'r' shown by dotted line.

Now the charge q is given by the relation

$$E = \frac{q}{\epsilon_0 A}.$$

$$\therefore q = E\epsilon_0 A.$$

But area of the Gaussian surface is given by

$$A = 2\pi r L.$$

$$\therefore q = E\epsilon_0 \times 2\pi r L$$

or $$\boxed{E = \frac{q}{2\pi r L \epsilon_0}} \quad \text{— (i)}$$

Now the potential difference is given by the relation

$$V = \int_{+}^{-} E\, ds \qquad \because ds = dr.$$

$$V = \frac{q}{2\pi r L\epsilon_0}\int_a^b dr$$

$$V = \frac{q}{2\pi L\epsilon_0}\int_a^b \frac{dr}{r}$$

$$V = \frac{q}{2\pi L\epsilon_0}\left| \ln r \right|_a^b.$$

$$V = \frac{q}{2\pi L\epsilon_0}(\ln b - \ln a)$$

$$\boxed{V = \frac{q}{2\pi L\epsilon_0} \ln \frac{b}{a}}$$

Now the capacitance of capacitor is

$$C = \frac{q}{V}$$

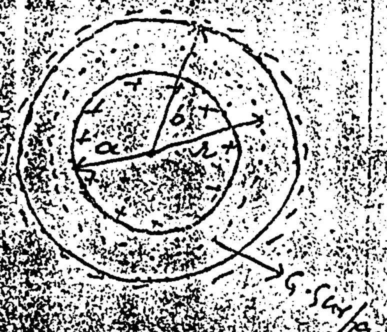
$$So \quad C = \frac{q}{\frac{q}{2\pi L\epsilon_0} \ln \frac{b}{a}}$$

$$\boxed{C = 2\pi\epsilon_0 \frac{L}{\ln b/a}}$$

This is the capacitance of cylinderical plate capacitor. It depends on geometry of the capacitor and the medium b/w the plates.

5- Capacitance of a Spherical Plate Capacitor:

A spherical capacitor consists of two concentric conducting spheres. Let a and b be their radii. We take Gaussian surface in the form of a sphere of radius r shown by the dotted line.



Now the charge q is given by the relation as

$$E = \frac{q}{\epsilon_0 A}$$

$$q = E\epsilon_0 A$$

But area of Gaussian surface is given by

$$A = 4\pi r^2$$

$$q = E\epsilon_0 4\pi r^2$$

$$E = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2}$$

Now the potential difference b/w the two spheres is given by

$$V = \int_{+}^{-} E\,ds = -\int_{a}^{b} E\,dr \qquad ds = -dr$$

$$V = \frac{q}{4\pi\epsilon_0} \int_{a}^{b} \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\epsilon_0} \int_a^b r^{-2}\, dr$$

$$= \frac{q}{4\pi\epsilon_0} \left| \frac{r^{-1}}{-1} \right|_a^b$$

$$= +\frac{q}{4\pi\epsilon_0} \left(-\frac{1}{r} \right)_a^b$$

$$= \frac{q}{4\pi\epsilon_0} \left(-\frac{1}{b} + \frac{1}{a} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{b-a}{ab} \right)$$

Now the capacitance is given by

$$C = \frac{q}{V}$$

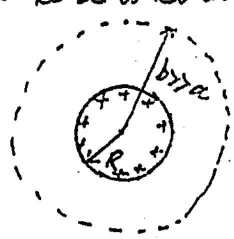
$$C = \frac{q}{\frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)}$$

$$\boxed{C = 4\pi\epsilon_0 \left(\frac{ab}{b-a} \right)}$$

This is the capacitance of a spherical plate capacitor.

5. Capacitance of an Isolated Sphere:

Consider a single isolated conductor in the form of a sphere of radius $a = R$. The other missing plate is supposed to be a conducting phere of infinite radius. So $b \to \infty$.



re charge will reside on the outer rface of isolated sphere. The charge ll behave as a point charge concentrated at the centre of the here. Therefore charge q is given by relation

$$E = \frac{q}{\epsilon_0 A}$$

$$q = E\epsilon_0 A.$$

Put area of sphere $A = 4\pi r^2$.

$$\therefore\ q = E\epsilon_0 \times 4\pi R^2.$$

$$\therefore\ E = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{R^2}.$$

Now potential difference is given by,

$$V = \int_{+}^{-} E\,ds.$$

$$V = \int_R^\infty E\,dr = \frac{q}{4\pi\epsilon_0}\int_R^\infty \frac{dR}{R^2}$$

$$= \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{R}\right)_R^\infty$$

$$= \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{\infty} + \frac{1}{R}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(0 + \frac{1}{R}\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{R}$$

Now the capacitance of sphere is given by

$$C = \frac{q}{V}$$

$$= q \times \frac{1}{V}.$$

$$= q \times \frac{4\pi\epsilon_0 R}{q}.$$

$$\boxed{C = 4\pi\epsilon_0 R}$$

This is the expression for capacitance of an isolated sphere.

Sample Problem-2

Sample Problem 2 The plates of a parallel-plate capacitor are separated by a distance $d = 1.0$ mm. What must be the plate area if the capacitance is to be 1.0 F?

Sol $d = 1.0\,mm = 10^{-3}\,m.$, $C = 1.0F$; $A = ?$

As $C = \epsilon_0 A/d$.

$$A = \frac{Cd}{\epsilon_0}$$

$$= \frac{1\times10^{-3}}{8.85\times10^{-12}} = \frac{1}{8.85}\times10^{9}.$$

$$= 0.11\times10^{9}.$$

$$A = 1.1\times10^{8}\,m^2 \quad \text{Ans}$$

Sample Problem 3

Sample Problem 3 The space between the conductors of a long coaxial cable, used to transmit TV signals, has an inner radius $a = 0.15$ mm and an outer radius $b = 2.1$ mm. What is the capacitance per unit length of this cable?

Sol:- $a = 0.15\,mm$, $b = 2.1\,mm$, $\frac{C}{L} = ?$

As $C = 2\pi\epsilon_0 \frac{L}{\ln(\frac{b}{a})}$.

$$\therefore \frac{C}{L} = \frac{2\pi\epsilon_0}{\ln\frac{b}{a}}$$

$$= \frac{2\times3.14\times8.85\times10^{-12}}{\ln(\frac{2.1}{0.15})}$$

$$= \frac{55.578}{2.639}\times10^{-12}$$

$$= 21\times10^{-12}\,F/m.$$

$$\boxed{\frac{C}{L} = 21\,pF/m} \quad \text{Ans.}$$

Sample Problem. 4

Sample Problem 4 What is the capacitance of the Earth, viewed as an isolated conducting sphere of radius 6370 km?

Sol:- $C = ?$, $R = 6370\,km = 6.37\times10^{6}\,m.$

As $C = 4\pi\epsilon_0 R$

$$= 4\times3.14\times8.85\times10^{-12}\times6.37\times10^{6}.$$

$$= 708\times10^{-6} = 7.08\times10^{-9}.$$

$$= 7.1\times10^{-9} = 710\times10^{-6}\,F.$$

$$\boxed{C = 710\,\mu F} \quad \text{Ans.}$$

Sample Problem-5

Sample Problem 5 (a) Find the equivalent capacitance of the combination shown in Fig. 7a. Assume

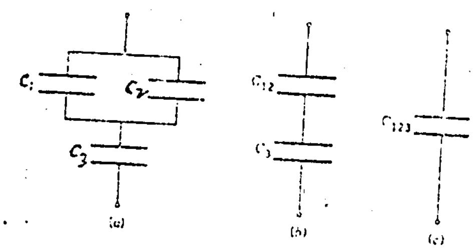
$C_1 = 12.0\,\mu F$, $C_2 = 5.3\,\mu F$, and $C_3 = 4.5\,\mu F$.

(b) A potential difference $V = 12.5$ V is applied to the terminals in Fig. 7a. What is the charge on C_1?

Sol :- $C_1 = 12\,\mu F$, $C_2 = 5.3\,\mu F$. $C_3 = 4.5\,\mu F$.

(a) $C_{123} = ?$.

Resultant of C_1, C_2 (∥ Comb.).



$$C_{12} = C_1 + C_2 .$$

$$C_{12} = 12 + 5.3 = 17.3\,\mu F .$$

Now resultant of C_{12}, C_3 (Series Comb.).

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} .$$

$$= \frac{1}{17.3} + \frac{1}{4.5}$$

$$C_{123} = \frac{4.5 + 17.3}{17.3 \times 4.5} =$$

$$C_{123} = \frac{(17.3)(4.5)}{21.8} = 3.57\,\mu F$$

$$\therefore \boxed{C_{123} = 3.57\,\mu F} \quad \text{Ans.}$$

(b) $V = 12.5$, charge on $C_1 = q_1 = ?$.

As C_{12} and C_3 are in series

$$\therefore q_{12} = q_3 = q_{123} .$$

Now $q_{123} = C_{123} V$.

$$= 3.57 \times 12.5$$

$$q_{123} = 44.6\,\mu C .$$

$$\therefore q_{12} = 44.6\,\mu C .$$

Now $q_{12} = C_{12} V_{12}$.

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6}{17.3}$$

$$\boxed{V_{12} = 2.58 \text{ Volts}}$$

Now C_1 and C_2 are in parallel

12.

$$\therefore \quad V_1 = V_2 = V_{12}$$

$$\therefore \quad q_1 = C_1 V_1$$

$$= 12 \times 2.58$$

$$\boxed{q_1 = 31 \mu C} \quad \text{Ans}$$

7 Energy Stored in an Electric Field:

We know that a charged capacitor possesses energy. So in charging a capacitor, the battery has to do work, this work done becomes the energy of the capacitor. This work is done by the battery because battery pulls the electrons from one plate and transfer them to the other plate.

So battery has to do work in seperating the equal and opposite charges. This energy is stored in the electric field between the plates as electric P.E. This energy can be recovered as K.E if the charges are allowed to come together.

Let us find the expression for energy of parallel plate capacitor. Suppose A is the area of each plate and d is the distance b/w them. Let the medium b/w the plates be free space.

Suppose charge q is transferred from one plate to the other. If V is the potential diff b/w the plates, then work done in transferring a small +ve charge dQ from -ve plate to +ve plate will be

$$dw = V\,dQ.$$

If C is the capacitance of the capacitor, then

$$V = \frac{Q}{C}.$$

$$\therefore dw = \frac{Q}{C}\,dQ.$$

Total work done in charging the capacitor is.

$$\int dw = \int_0^Q \frac{Q}{C}\,dQ.$$

$$\text{But} \quad w = \int_0^Q \frac{Q}{C}\,dQ.$$

This work becomes the P.E 'U' of the capacitor.

$$\therefore \quad U = \int_0^Q \frac{Q}{C}\, dQ.$$

$$U = \frac{1}{C} \int_0^Q Q\, dQ.$$

$$= \frac{1}{C} \left| \frac{Q^2}{2} \right|_0^Q.$$

$$U = \frac{Q^2}{2C} \qquad \text{But } Q = CV.$$

$$\therefore U = \frac{C^2V^2}{2C}$$

$$\boxed{U = \frac{1}{2} CV^2} \quad - (1)$$

This is the energy stored in a charged capacitor.

Energy Density

"The energy per unit volume of the capacitor is called energy density."

$$\therefore \text{ Energy density} = \frac{\text{Energy}}{\text{Volume}}.$$

If we denote the energy density by 'u'.

Then. $u' = \dfrac{u}{\text{Vol.}}$

But Volume = Area × distance

$= A \times d$

$$\therefore u' = \frac{u}{A \times d}$$

Putting u from (1) we get.

$$u' = \frac{1}{2} CV^2 \times \frac{1}{Ad}.$$

$$u' = \frac{CV^2}{2Ad}.$$

But for a parallel plate capacitor

$$C = \frac{\epsilon_0 A}{d}.$$

$$\therefore u' = \frac{\epsilon_0 A}{d} \quad \frac{V^2}{2Ad}$$

$$u' = \frac{\epsilon_0 V^2}{2d^2}$$

But $V = E d$.

$$\therefore \quad u' = \frac{\epsilon_0}{2d^2}(E^2 d^2)$$

$$u' = \frac{\epsilon_0 E^2}{2}$$

$$u' = \frac{1}{2}\epsilon_0 E^2.$$

This is the expression for energy density of the capacitor.

Sample Problem - 7

Sample Problem 7 An isolated conducting sphere whose radius R is 6.85 cm carries a charge $q = 1.25$ nC. (a) How much energy is stored in the electric field of this charged conductor? (b) What is the energy density at the surface of the sphere? (c) What is the radius R_0 of a spherical surface such that one-half of the stored potential energy lies within it?

Sol. Radius $= R = 6.85\text{ cm} = 0.0685\text{ m}$, $q = 1.25\text{ nC} = 1.25 \times 10^{-9}\text{ C}$.

(a) Energy stored $= ?$

As energy stored is $u = \frac{1}{2}CV^2$.

$$= \frac{1}{2} C \frac{q^2}{C^2} \qquad \because V = q/C.$$

$$u = \frac{1}{2}\frac{q^2}{C}.$$

$\because C = 4\pi\epsilon_0 R$ for isolated sphere.

$$\therefore u = \frac{q^2}{2 \times 4\pi\epsilon_0 R} = \frac{q^2}{8\pi\epsilon_0 R}.$$

Putting values of q and R we get,

$$u = \frac{(1.25 \times 10^{-9})^2}{8 \times 3.14 \times 8.85 \times 10^{-12} \times 0.0685}$$

$$= \frac{1.25 \times 1.25}{15.23} \times 10^{-18+12}$$

$$= 0.10259 \times 10^{-6}.$$

$$= 1.0259 \times 10^{-7} = 1.03 \times 10^{-7}$$

$$= 103 \times 10^{-9}\text{ J}$$

$$= 103\text{ nJ}$$

$\boxed{u = 103\text{ nJ}}$ Ans.

(b). Energy density at the surface of the sphere = ?

As energy density is given by,

$$u = \frac{1}{2}\epsilon_0 E^2$$

Now $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$.

$$\therefore u = \frac{1}{2}\epsilon_0\left(\frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\right)^2.$$

$$= \frac{1}{2}\epsilon_0 \times \frac{q^2}{16\pi^2\epsilon_0^2 R^4}$$

$$u = \frac{q^2}{32\pi^2\epsilon_0 R^4}$$

Putting the values we get

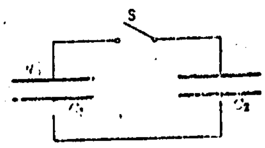
$$u = \frac{(1.25\times 10^{-9})^2}{32\times(3.14)^2\times 8.85\times 10^{-12}\times(0.0685)^2}$$

$$= \frac{1.25\times 1.25\times 10^{-18}\times 10^{12}}{0.0615}$$

$$\boxed{u = 25.4 \quad \mu J/m^3}$$ Ans.

Sample Problem-6

Sample Problem 6 A 3.55-μF capacitor C_1 is charged to a potential difference $V_0 = 6.30$ V, using a battery. The charging battery is then removed, and the capacitor is connected as in Fig. 9 to an uncharged 8.95-μF capacitor C_2. After the switch S is closed, charge flows from C_1 to C_2 until an equilibrium is established, with both capacitors at the same potential difference V. (*a*) What is this common potential difference? (*b*) What is the energy stored in the electric field before and after the switch S in Fig. 9 is thrown?



Sol: $C_1 = 3.35\ \mu F$.

$V_0 = 6.30V$, $C_2 = 8.95\mu F$.

(a) $V = ?$

The original charge q_0 on C_1 is given by

$$q_0 = C_1 V_0 .$$

Now C_1 is connected in ∥ with C_2. Charge q_0 is divided into two parts q_1 and q_2.

$$\therefore q_0 = q_1 + q_2 .$$

$$\therefore C_1 V_0 = C_1 V + C_2 V$$

But $q_1 = C_1 V$

$q_2 = C_2 V$.

$$C_1 V_0 = V(C_1 + C_2).$$

$$\boxed{V = \frac{C_1 V_0}{C_1 + C_2}}$$

Putting the values of C_1, V_0, C_2 we get.

$$V = \frac{3.56 \times 6.30}{3.55 + 8.95} = \frac{22.365}{12.5}.$$

$$\boxed{V = 1.79 \text{ Volts}}$$ Ans

(b) Energy stored = ? before and after the switch S is thrown.

The initial stored energy is

$$U_i = \frac{1}{2} C_1 V_0^2 .$$

$$= \frac{1}{2} \times 3.35 \times 10^{-6} \times (6.30)^2 .$$

$$U_i = 70.45 \times 10^{-6} J.$$

$$\boxed{U_i = 70.5\ \mu J}$$ Ans

The final energy after the switch S is thrown is

$$U_f = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 .$$

$$= \frac{1}{2} V^2 (C_1 + C_2)$$

$$= \frac{1}{2} (1.79)^2 \left(3.55 \times 10^{-6} + 8.95 \times 10^{-6}\right).$$

$= 1.6025\,(12.5 \times 10^{-6})$.

$= 20.02\ \mu J$

$\boxed{U_f = 20.0\ \mu J}$ Ans.

$\therefore U_f < U_i$ Because some energy is appears as thermal energy in the connecting wires.

8. Capacitors with Dielectric:

"An insulating material exerted b/w the plates of a capacitor is called dielectric."

e.g glass, paper, plastic etc.

It is found that capacitance of a capacitor increases in the presence of dielectric b/w the plates. The presence of dielectric changes the electric field b/w the plates.

Michell Faraday in 1837 first observed the effect of placing dielectric b/w the plates. Farday took two similar capacitors, filling one with dielectric and other with air under normal conditions. He charged both the capacitors to bothe same potentiae difference. He observed that the charge on the capacitor with the dielectric was greater than that on the other. So by the eq. $C = \frac{Q}{V}$, it is clear that capacitance of capacitor increases in the presence of dielectric.

If C_o is the capacitance of a capacitor with air as medium b/w the plates and C is the capacitance with dielectric as medium b/w the plates then $C > C_o$ by a factor K_e called dielectric constant. i.e the ratio $\frac{C}{C_o}$ is called dielectric constant.

$$\therefore K_e = \frac{C}{C_o}.$$

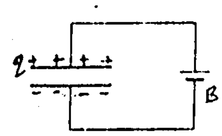
So dielectric constant may be defined as.

"The ratio of capacitance of a capacitor with dielectric as medium to the capacitance of the same capacitor with air as medium b/w the plates."

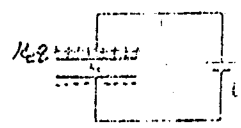
The dielectric constant is independent of size and shape of conductor.

Let us discuss the effect of dielectric b/w the plates with and with out plates battery.

(i) Battery Connected



(a)



(b)

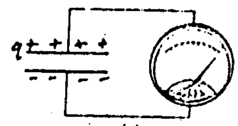
Fig.(i) shows a battery which charges the capacitor with charge q. As the battery remains connected, so potential difference V and electric field both remain constant.

Now when the dielectric is placed b/w the plates as shown in fig.(ii) then charge increases by the factor K_e. So now charge has value $= K_e q$.

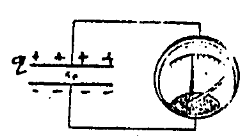
So the additional charge $K_e q - q = q(K_e - 1)$ is moved from -ve to +ve plate by the battery as the dielectric is placed b/w the plates.

In this case potential difference remains the same while the charge on the capacitor increases. So capacitance increases due to increase in charge.

(ii) Battery Disconnected



(a)



(b)

In fig(iii) battery is disconnected after the capacitor is charged to charge 'q'.

Now when dielectric is placed as shown in fig (iv) then the charge remains constt between potential difference decreases by the factor K_e i.e potential difference decreases from V to $\frac{V}{K_e}$.

So by the equation $q = CV$. if we decreases by the factor K_e then C must increase by the same factor K_e so that q remains constant.

So when battery is disconnected, the charge remains constant but potential difference decreases. So capacitance increases due to decrease in potential difference.

So in both the cases capacitance increases in the presence of dielectric.

For a parallel plate capacitor, for free space is

$$C_0 = \frac{\epsilon_0 A}{d}$$

In the presence of dielectric the capacitance becomes

$$C = K_e C_0.$$

$$C = \frac{K_e \times \epsilon_0 A}{d}$$

Similarly for a point charge q embeded in a dielectric, the electric field is

$$E = \frac{1}{4\pi\epsilon_0 K_e} \frac{q}{r^2}.$$

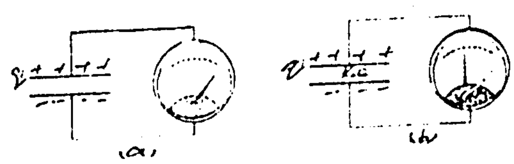
It should be noted that the effect of K_e is to decrease the electric field.

Sample Problem-8

Sample Problem 8 A parallel-plate capacitor whose capacitance C_0 is 13.5 pF has a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected and a porcelain slab ($\kappa_e = 6.5$) is slipped between the plates as in Fig. 11b. What is the stored energy of the unit, both before and after the slab is introduced?

Sol. $C_0 = 13.5\,pF = 13.5 \times 10^{-12} F.$, $V = 12.5$ volt, $K_e = 6.5$.

After disconnecting the battery, what is stored energy before and after placing dielectric?



The initial energy is given by

$$U_i = \frac{1}{2} C_0 V^2$$

$$= \frac{1}{2} \times (13.5 \times 10^{-12}) (12.5)^2.$$

$$= 1055 \times 10^{-12} J.$$

$$\boxed{U_i = 1055\, pJ}$$ Ans

Now we find energy when dielectric is placed is given by replacing C_0 by C.

$$U_f = \frac{1}{2} C V^2.$$

$$= \frac{1}{2} C \frac{q^2}{C^2}$$

$$U_f = \frac{q^2}{2C}$$

Now $C = K_e C_0$.

$$= \frac{q^2}{2K_e C_0}$$

But $\frac{q^2}{2C_0} = U_i$.

$$= \frac{q^2}{2C_0} \times \frac{1}{K_e}.$$

$$U_f = \frac{U_i}{K_e}.$$

$$= \frac{1055}{6.5}$$

$$\boxed{U_f = 162\ pJ}$$ Ans.

Dielectrics ; An atomic view:

An insulating material exerted b/w the plates of a capacitor is called dielectric.

Types of Dielectric: There are two types of dielectrics

(i) Polar diedectrics. (ii) Non-polar diedectrics.

Now we study the effect of external electric field on polar and non-polar diedectrics one by one.

Effect of External Field on Polar Dielectric:

"The dielectrics whose molecules have permanent electric dipole moments are called polar dielectrics."

The centres of +ve and -ve charges in an atom do not coincide. e.g molecules like H_2O, N_2O are polar.

When polar dielectrics are placed in an external electric field, the electric dipole moments tend to a align themselves in the direction of external electric field. But due to thermal agitation, the degree of alignment is not complete. These dipole moments align themselves more and more as the external electric field is increased or temperature is decreased.

In the absence of external electric field, these dipole moments are randomly oriented. The following figs the orientation of dipole moments of a polar dielectric in the absence and presence of external electric field.

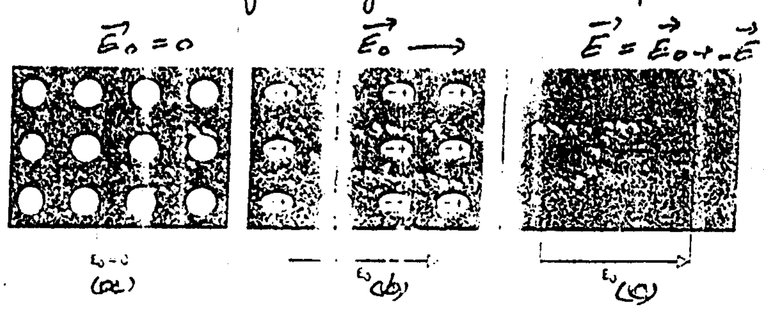
Non-Polar Dielectrics:

The dielectrics whose molecules do not possess permanent dipole moments are called non-polar dielectrics.

e.g. molecules like H_2, N_2, O_2 are non-polar. Non-polar dielectrics can have dipole moment in the presence of external electric field. We know that in a dielectric the charges are static. But when the dielectric is subjected to an external electric field as in b/w the plates of a charged capacitor, the +ve and -ve charges in an atom are displaced away from each other due to electrostatic forces. So the +ve and -ve charges in a molecule are displaced away from their equilibrium positions. So each atom becomes an electric dipole.

So the whole dielectric is divided into dipoles when placed b/w the plates of a charged capacitor. The dielectric is then said to be

The dielectric is then said to be polarized and the phenomenon is called polarization of dielectric.
Let us take a parallel plate capacitor having charge q and not connected to a battery.
Now consider a slab of dielectric placed at a place where there is no external electric field. Fig.(a) shows the spherical atoms of dielectric.



(a) (b) (c)

in the absence of external electric field the centres of +ve and -ve charge coincide.

Fig.(b) shows the same dielectric placed b/w the plates of charged capacitor. So the +ve and -ve charges are displaced away from their equilibrium position. Due to this polarization opposite charges induce near the plates. The induced charges produce an electric field $\vec{E}'$ opposite to external electric field. So in the interior of dielectric the electric field $\vec{E} = \vec{E}_0 + (-\vec{E}')$. The direction of this net electric field $\vec{E}$ is same as that of $\vec{E}_0$ but smaller in magnitude.

Hence when we place a dielectric in an electric field, the induced field decreases the original external electric field within the dielectric.

Gauss's Law in Dielectrics

Consider a parallel plate capacitor with free space b/w the plates. Then Gauss's law can be written as,

$$\epsilon_0 \oint \vec{E} \cdot \vec{dA} = q.$$

$$\epsilon_0 E_0 A = q.$$

$$E_0 = \frac{q}{\epsilon_0 A} \quad — ①$$

When the dielectric is placed b/w the plates, the charge q remains the same. However due to polarization of dielectric opposite charges appear near the plates which are called induce charges.

Let -q' be the induced charge near the +ve plate. So net charge on the Gaussian is q-q'.

So Gauss's law becomes

$$\epsilon_0 \oint \vec{E}.d\vec{A} = q - q' \quad — ②$$

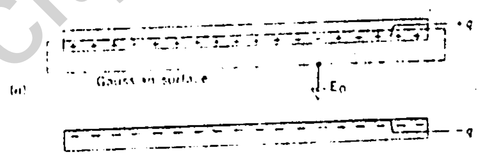
$$\epsilon_0 E A = q - q'.$$

$$E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad — ③$$

where E is the field in the presence of dielectric. The two charges q and -q' lie within the Gaussian surface.

Now the dielectric decreases the field by the factor K_e.

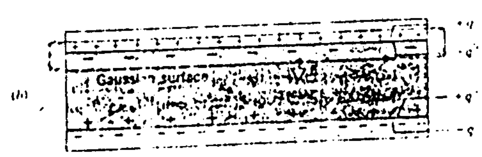
$$\therefore E = \frac{E_0}{K_e}.$$



Putting value of E_0 from ①.

$$E = \frac{q}{\epsilon_0 A} \times \frac{1}{K_e} \quad — ④.$$

Comparing ③ and ④ we get.



$$\frac{q}{\epsilon_0 A}\frac{1}{K_e} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}.$$

$$\frac{q}{K_e} = q - q'.$$

$$\text{or } q' = q - \frac{q}{K_e}.$$

$$q' = q\left(1 - \frac{1}{K_e}\right) \quad — ⑤.$$

Equation ④ shows that $q' < q$ in magnitude and $q' = 0$ if no dielectric is present i.e $K_e = 1$.

Now putting the value of q' from ⑤ in ② we get Gauss's law in dielectric as

$$\epsilon_0 \oint \vec{E}\, d\vec{A} = q - q\left(1 - \frac{1}{K_e}\right).$$

$$= q - q + \frac{q}{K_e}$$

$$\epsilon_0 \oint E \cdot dA = \frac{q}{K_e}$$

or $$\epsilon_0 \oint K_e \vec{E} \cdot \vec{dA} = q.$$

This is called Gauss's law in dielectrics.

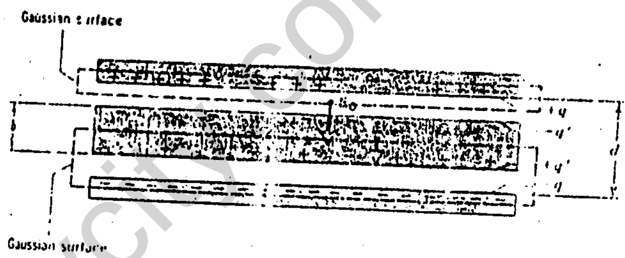
Sample Problem: 9

Sample Problem 9 Figure 16 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ_e is placed between the plates as shown. Assume

$A = 115$ cm², $d = 1.24$ cm, $b = 0.78$ cm,

$\kappa_e = 2.61$, $V_0 = 8$. V.

(a) What is the capacitance C_0 before the slab is inserted? (b) What free charge appears on the plates? (c) What is the electric field E_0 in the gaps between the plates and the dielectric slab? (d) Calculate the electric field E in the dielectric slab. (e) What is the potential difference between the plates after the slab has been introduced? (f) What is the capacitance with the slab in place?



Sol:- $A = 115\,cm^2 = 115 \times 10^{-4} m^2$.

$d = 1.24\,cm = 1.24 \times 10^{-2} m$

Thickness of slab $= b = 0.78\,cm = .78 \times 10^{-2} m$.

$K_e = 2.61$, $V_0 = 85$ Volt.

(a) Capacitance befor the slab is inserted $= C_0 = ?$

As $C_0 = \frac{\epsilon_0 A}{d}$

$$= \frac{8.85 \times 10^{-12} \times 115 \times 10^{-4}}{1.24 \times 10^{-2}} = \frac{8.85 \times 115}{1.24} \times 10^{-14}.$$

$$= 820.76 \times 10^{-14} = 821 \times 10^{-14}.$$

$$= 8.21 \times 10^{-12}$$

$\boxed{C_0 = 8.21\ PF}$ Ans.

(b) Free charge on the plates $= q = ?$

As $q = C_0 V_0$.

$$= 8.21 \times 10^{-12} \times 85.5 = 702 \times 10^{-12} C$$

$\boxed{q = 702\ PC}$ Ans.

(c) Electric field in the gaps b/w the plates and dielectric slab $= E_0 = ?$

As $\epsilon_0 E_0 A = q$.

$$E_0 = \frac{q}{\epsilon_0 A}$$

$$= \frac{702 \times 10^{-12}}{8.85 \times 10^{-12} \times 115 \times 10^{-4}}$$

$$= \frac{702}{8.85 \times 115} \times 10^4 = 0.6897 \times 10^1.$$

$$= 68.97 \times 10^2$$

$$= 69 \times 10^2$$

$$= 6900 \text{ V/m}$$

$$= 6.9 \times 10^3 \text{ V/m}$$

$$\boxed{E_0 = 6.9 \text{ kV/m}} \quad \text{Ans}$$

(d) Electric field E in the dielectric slab = ?

As $\epsilon_0 E A = \frac{q}{K_e}$.

$$\therefore \; E = \frac{q}{K_e \epsilon_0 A}.$$

As from (c)

$$E = \frac{E_0}{K_e}.$$

$$= \frac{6.9 \times 10^3}{2.61}$$

$$= 2.64 \times 10^3 \text{ V/m}$$

$$\boxed{E = 2.64 \text{ kV/m}} \quad \text{Ans}$$

(e) Pot. difference V after the dielectric is placed = ?

$$V = \int_{+}^{-} E \, ds.$$

$$= E_0 (d - b) + E b.$$

$$= 6900 \times 10^{-2} (1.24 \times 10^{-2} - 0.78 \times 10^{-2}) + 2.64 \times 0.78 \times 10$$

$$= 69 \times 0.46 + 2.64 \times 7.8.$$

$$= 31.74 + 20.592. = 52.332.$$

$$\boxed{V = 52.3 \text{ volt}} \quad \text{Ans}$$

(f) Capacitance in the presence of dielectric = C = ?

As $C = \frac{q}{V}$

$$= \frac{702 \times 10^{-12}}{52.3}$$

$$= 13.4 \times 10^{-12} F.$$

$\boxed{C = 13.4 PF}$ Ans

-: The End :-

Nasir Pervaiz Butt
M. Sc. (Physics) M. Phil
Assistant Professor
Govt. College Sargodha

House No. 338 St. No. 7
Cheema Colony,
Sargodha.

Ch. 32

ELECTRIC CURRENT

1. Electric Current:

"Rate of flow of charge through a conductor is called electric current."

or "The charge flowing per second through any section of the conductor is called electric current."

Suppose a charge dq flows through any section of the conductor in time dt. Then current in the conductor is

$$i = \frac{dq}{dt} \quad \text{--- (1).}$$

Unit of Current: The S.I unit of current is Ampere.

It is given as

$$1\,\text{Amp} = \frac{1\,\text{coulomb}}{1\,\text{Second}}.$$

"The current is said to be one Ampere if one coulomb charge flows through any section of a conductor in one second."

The net charge passing through the surface in any time interval is obtained from (1) as

$$\int dq = \int i\,dt.$$

$$q = \int i\,dt.$$

If current is constant in time then

$$q = i\int dt.$$

$$q = it$$

$$i = q/t$$

The electric current i is the same at all points of a circuit irrespective of the cross sectional area at different points.
A +ve charge moving in one direction is equivalent to a -ve charge moving in the opposite direction. Hence for simplicity we adopt the following convention.

"The direction of current is the direction in which +ve charge moves even if the actual current is due to -ve charge."

The current due to motion of electrons is called electronic current and current due to motion of +ve charges is called Conventional current. There is no experimental difference b/w electronic current and conventional current. So both electronic and conventional current are equivalent. Current is a scalar quantity because it does not obey laws of vector addition.

---

2. Current Density:

It is defined as,

"Current flowing per unit area held ⊥ to the direction of flow of current."

It is a vector quantity. It is denoted by $\vec{J}$
If we have a conductor of uniform area of cross section 'A' and the current i is uniformly distributed across the conductor, then the magnitude of current density is given by,

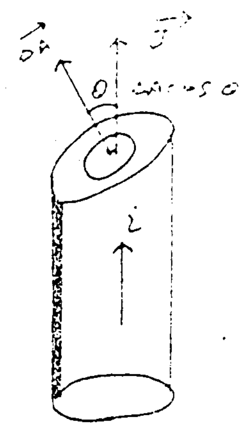
$$J = \frac{i}{A}$$

Current is a macroscopic quantity while current density is microscopic quantity.
The direction of $\vec{J}$ at any point is the direction in which +ve charge ... that point. Its S.I unit is Amp/m^2.

us consider small current flowing normally through an area ΔA. Then current density is

$$J = \lim_{\Delta A \to 0} \frac{\Delta i}{\Delta A} = \frac{di}{dA}.$$

If the plane of area makes an angle θ with the direction of flow of current, then



$$J = \lim_{\Delta A \to 0} \frac{\Delta i}{\Delta A \cos\theta} = \frac{di}{dA\cos\theta}$$

$$di = J\,dA\cos\theta.$$

$$di = \vec{J}\cdot\vec{dA}.$$

Then the total current is

$$\int di = \int \vec{J}\cdot\vec{dA}$$

$$i = \int \vec{J}\cdot dA$$

$$i = \int J\,dA\cos\theta$$

If the plane of area is $\perp$ to flow of current, then $\theta = 0°$

$$i = \int J\,dA\cos 0°$$

$$i = \int J\,dA$$

If the current density is uniform over the whole area then

$$i = J\int dA.$$

$$i = JA.$$ where A is the area of cross section of the conductor.

Thus $J = \frac{i}{A}.$

Relation b/w Drift Velocity and Current Density

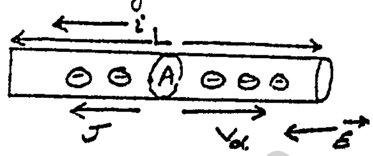
"The constant average velocity with which the electrons move (drift) in a conductor under the action of an applied electronic field is called average drift velocity" and is denoted by V_d.

Let us find the relation b/w drift speed V_d and current density $\vec{J}$

Consider a conductor of length L and area of cross section A. Then volume of the conductor is $= AL$.

Let n be the number of conduction electrons per unit volume in the conductor.

Then total number of electrons in length L is $= nAL$. If e is the charge one one electron then total charge on these electrons $= nALe$.



$$\therefore q = nALe.$$

If the electrons cover this length L in t sec, then magnitude of average drift velocity is given by.

$$V_d = \frac{L}{t}$$

$$\therefore t = \frac{L}{V_d}$$

Then current i is given by,

$$i = \frac{q}{t}$$

$$= \frac{nALe}{L/V_d} = \frac{nALeV_d}{L}$$

$$i = nAeV_d.$$

Now current density J is given by

$$J = \frac{i}{A}$$

$$= \frac{nAeV_d}{A}$$

$$\boxed{J = neV_d}$$

This is the relation of current density and drift velocity. Since $\vec{J}$ and $\vec{V}_d$ both are vectors in opposite directions, then the above relation in vector form becomes

$$\vec{J} = -ne\vec{V}_d.$$

Sample Problem: 1

Sample Problem 1 One end of an aluminum wire [illegible] diameter is 2.5 mm is welded [illegible] end of a copper wire whose

[illegible] current [illegible]

Sol:- $d_{Al} = 2.5\,mm.$

$$r_{Al} = \frac{2.5}{2} = 1.25\,mm = 1.25 \times 10^{-3}\,m.$$

$$d_{cu} = 1.8\,mm, \quad r_{cu} = \frac{1.8}{2} = 0.9\,mm = 0.9 \times 10^{-3}$$

current $i = 1.3\,Amp.$

(i) current density in Al wire $= J_{Al} = ?$

(ii) current density in Cu wire $= J_{cu} = ?$

Al $\quad J_{Al} = \dfrac{i}{A_{Al}} = \dfrac{i}{\pi r_{Al}^2} = \dfrac{1.3}{3.14 \times (1.25 \times 10^{-3})^2}$

$$J_{Al} = \frac{1.3 \times 10^6}{4.90625}.$$

$$J_{Al} = 0.26 \times 10^6\,Amp/m^2.$$

$$= 0.26 \times 10^6 \frac{Amp}{(10^2\,cm)^2} = 0.26 \times 10^6 \frac{Amp}{10^4\,cm^2}.$$

$$= 0.26 \times 10^2\,Amp/cm^2.$$

$$\boxed{J_{Al} = 26\,A/cm^2} \text{ Ans}$$

Now $\quad J_{cu} = \dfrac{i}{A_{cu}} = \dfrac{i}{\pi r_{cu}^2} = \dfrac{1.3}{3.14 \times (0.9 \times 10^{-3})^2}$

$$J_{cu} = \frac{1.3 \times 10^6}{2.5434}. = 0.51 \times 10^6\,A/m^2.$$

$$= 0.51 \times 10^6 \frac{A}{(10^2\,cm)^2} = 0.51 \frac{\times 10^6\,A}{10^4\,cm^2}.$$

$$= 0.51 \times 10^2\,A/cm^2$$

$$\boxed{J_{cu} = 51\,A/cm^2} \text{ Ans.}$$

Sample Problem 3

Sample Problem 3 A strip of silicon, of width $w = 3.2$ mm a[illegible] thickness $d = 250\,\mu m$, carries a current i of [illegible]. The s[illegible] is an *n-type semiconductor*, having been "doped" with a [illegible] trolled amount of phosphorus impurity. The doping has [illegible] effect of greatly increasing n, the number of charge carriers (ele[illegible] trons, in this case) per unit volume, as compared with the va[illegible] for pure silicon. In this case, $n = 8.0 \times 10^{21}\,m^{-3}$. (a) What [illegible] current density in the strip? (b) What is the drift speed?

Sol:- Width $= w = 3.2\,mm = 3.2 \times 10^{-3}\,m.$

Thickness $= d = 250\,\mu m = 250 \times 10^{-6}\,m$

current $= i = 190\,mA = 190 \times 10^{-3}\,Amp.$

number of electrons per unit volume $= n = 8 \times 10^{21}/m^3.$

(a) $J = ?$ (b) $V_d = ?$

As $J = \frac{i}{Area} = \frac{i}{wd}$.

$= \frac{190 \times 10^{-3}}{3.2 \times 10^{-3} \times 250 \times 10^{-6}}$

$= \frac{190}{3.2 \times 250} \times 10^{6}. = 0.2375 \times 10^{6}.$

$= 0.2375 \times 10^{6}. = 0.24 \times 10^{6}$

$\boxed{J = 2.4 \times 10^{5} \ Amp/m^{2}}$ Ans

(b) As $J = neV_d$.

$\therefore V_d = \frac{J}{ne}$

$= \frac{2.4 \times 10^{5}}{8 \times 10^{21} \times 1.6 \times 10^{-19}} = \frac{2.4}{8 \times 1.6} \times 10^{3}$

$= 0.1875 \times 10^{3} = 0.19 \times 10^{3}$

$\boxed{V_d = 190 \ m/s}$ Ans

3. Resistante, Resistivity & Conductivity:

Resistance "It is the opposition offered by the atoms of the conductor to the motion of free electrons."

If we apply the same potential difference to similar rods of wood and copper then different amounts of current flow in them. The currents are different due to different resistances of wood and Cu. If a potential difference V is applied b/w two ends of a conductor, then current flowing is proportional to the potential difference

i.e $V \propto i$.

$V = iR$.

or $R = V/i$

Where R is a constant of propotionality called resistance of the conductor. Its value depends on nature, temperature and dimensions of the conductor.

Unit of Resistance: The S.I unit of resistance is ohm. It is given as

$$1\,\Omega = \frac{1\,\text{Volt}}{1\,\text{Amp}}.$$

"The resistance of a conductor is one ohm if a potential difference of one volt across the conductor produces a current of one ampere in it.".

Resistivity: The resistance 'R' of a conductor of uniform area of cross section depends on its length L and area of cross section A as

$$R \propto L.$$

$$R \propto \frac{1}{A}.$$

$$R \propto \frac{L}{A}.$$

$$R = \frac{\rho L}{A}.$$

$$\rho = \frac{RA}{L}.$$

Here ρ is the constant of propotionality and is called resistivity. It is a characteristic of material rather than a particular specimen of the material. It is independent of dimensions and depends on nature and temperature only. It is also called specific resistance.

If $A = 1\,m^2$.

$L = 1\,m.$

Then $\rho = R.$

So it is defined as

"Resistance of a meter cube of a substance"

or It is the resistance [illegible]

1m and area of cross section.

Unit of Resistivity: The S.I unit of resistivity is Ohm-meter (Ω-m). It is defined from

$$\rho = \frac{RA}{L} \text{ as.}$$

$$1\,\Omega\text{-m} = \frac{\Omega\text{-m}^2}{m} = \Omega\text{-m}.$$

Conductivity:

"It is the reciprocal of resistivity"

It is denoted by σ

$$\therefore \sigma = \frac{1}{\rho}.$$

Unit of Conductivity:

The S.I unit of resistivity is $\frac{1}{\text{ohm-m}} = (\Omega\text{-m})^{-1}$

$$= \text{mho-m}^{-1}.$$

Relation b/w Conductivity & Current-density:

We know that potential difference across a conductor sets up an electric field in it. If V is the potential difference and E is the electric field then

$$V = EL \quad -(i)$$

But $V = IR$.

$$\therefore EL = IR$$

$$\therefore EL = I\frac{\rho L}{A}. \qquad \because R = \frac{\rho L}{A}.$$

$$E = \frac{I\rho}{A}.$$

$$E = \rho\left(\frac{I}{A}\right).$$

$$E = \rho J. \qquad \text{But } \frac{I}{A} = J.$$

$$J = \frac{E}{\rho}$$

$$J = \frac{1}{\rho}E.$$

$$\therefore J = \sigma E. \qquad \text{But } \frac{1}{\rho} = \sigma$$

In vector form $\boxed{\vec{J} = \sigma\vec{E}}$

This is the relation b/w conductivity and current density. It is also called microscopic form of Ohm's law
This shows that current density is directly proportional to electric intensity.

Sample Problem: 2

Sample Problem 2 What is the drift speed of the conduction electrons in the copper wire of Sample Problem 1?

From S.P 1
$J_{cu} = 0.51 \times 10^{6}\ A/m^2$

<u>Sol:</u> $V_d = ?$; $e = 1.6 \times 10^{-19} C$
$M = 63.5 \times 10^{-3}\ kg/mol$
$d = 8.96 \times 10^{3}\ kg/m^3$

As $J = neV_d$

$$V_d = \frac{J}{ne}$$

$$V_d = \frac{0.51 \times 10^{6}}{n \times 1.6 \times 10^{-19}} \quad \text{—①}$$

Let V be the volume of one mole of Cu.

∴ number of electrons per unit vol is given as

$$n = \frac{N_A}{V}$$

$$n = \frac{6.02 \times 10^{23}}{\frac{M}{d}} \qquad \because V = \frac{M}{d}$$

$$= \frac{6.02 \times 10^{23} \times d}{M}$$

$$= \frac{6.02 \times 10^{23} \times 8.96 \times 10^{3}}{63.5 \times 10^{-3}}$$

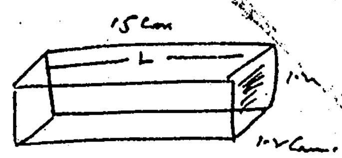
$$n = 8.49 \times 10^{28}\ \text{electrons}/m^3$$

putting the value of n in ① we get.

$$V_d = \frac{0.51 \times 10^{6}}{8.49 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$\boxed{V_d = 3.8 \times 10^{-5} m/s}\ \text{Ans.}$$

Sample Problem. 4



Sample Problem 4 A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. (a) What is the resistance of the block measured between the two square ends? (b) What is the resistance between two opposing rectangular faces? The resistivity of iron at room temperature is $9.68 \times 10^{-8}\ \Omega\cdot m$.

Sol: $\rho = 9.68 \times 10^{-8}\ \Omega\text{-}m$, $L = 15\,cm = 0.15\,m$.

(a) Resistance of block b/w square ends = ?

Area of square End $= 1.2 \times 1.2 = 1.44\,cm^2 = 1.44 \times 10^{-4}\,m^2$.

$$\therefore R = \frac{\rho L}{A}$$

$$= \frac{9.68 \times 10^{-8} \times 0.15}{1.44 \times 10^{-4}} = \frac{9.68 \times 0.15}{1.44} \times 10^{-4}.$$

$$= 1.0 \times 10^{-4}\,\Omega = 100 \times 10^{-6}\,\Omega$$

$\boxed{\Omega = 100\,\mu m}$ Ans.

(b) Resistance of two opposite rectangular faces = ?

Area of rectangular face $= 15 \times 1.2 = 18.0\,cm^2$.

$$A = 18 \times 10^{-4}\,m^2.$$

$$\text{As } R = \frac{\rho L}{A} = \frac{9.68 \times 10^{-8} \times 1.2 \times 10^{-2}}{18 \times 10^{-4}}$$

$$= \frac{9.68 \times 1.2}{18} \times 10^{-6} = 0.65 \times 10^{-6}$$

$\boxed{R = 0.65\,\mu\Omega}$ Ans

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4. Microscopic & Macroscopic Quantities:

V, I and R are macroscopic quantities. The corresponding microscopic quantities are $\vec{E}$, $\vec{J}$ and ρ or σ. $\vec{E}$, $\vec{J}$ and ρ or σ have values at every point of body.

The macroscopic quantities are related as $V = IR$.

The microscopic quantities are related as

$$\rho = \frac{E}{J} \quad \text{—(i)}$$

$$E = \rho J. \quad \text{—(ii)}$$

$$J = \sigma E \quad \text{—(iii)}.$$

he macroscopic quantities can be found by integrating over the microscopic quantities.

e.g $i = \int \vec{J} \cdot d\vec{A}$ — (A)

$V = \int \vec{E} \cdot d\vec{s}$ — (B)

In eq. (A) integral is a surface integral while in eq. (B) integral is a li integral.

Resistance R can be found by dividing (B) by (A)

i.e $R = \frac{V}{i}$.

$$R \quad \frac{\int \vec{E} \cdot d\vec{s}}{\int \vec{J} \cdot d\vec{B}}$$

If the conductor is a long wire of length L and area of cross section 'A'. Then the above relation becomes

$$R = \frac{EL}{JA}. \qquad \text{But } \frac{E}{J} = \rho$$

$$\therefore \ R = \frac{\rho L}{A}$$

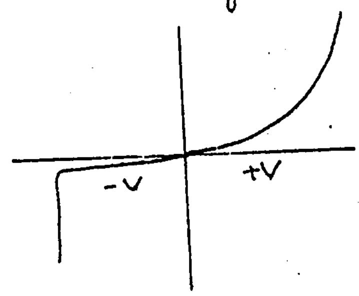
The macroscopic quantities V, i and R are important when we measure electrical quantities in real conductors. Their values are indicated on voltmeter, Ammeter and ohm meter. The microscopic quantities E, ρ and J are important when we are concerned with behaviour of matter rather than specimens of matter. The microscopic quantities are also important while studying the interior behaviour of irregular shaped conducting bodies.

5. Ohm's Law:

"A conducting device obeys Ohm's law if the resistance b/w any pair of points is independent of magnitude and polarity of the applied pot. difference"

A material or a circuit that obeys Ohm's law is called ohmic material.

Some electronic devices like Pn-junction does not obey Ohm's law. In a Pn-junction the current does not increase linearly with the voltage. It is also clear from fig. that behaviour of Pn-junction for +ve voltage is different than at -ve voltage.



It should be noted that the relation $V = iR$ is not a statement of Ohm's law. Acceleration to this relation $R = \frac{V}{I}$. So from this relation we must get a general definition of resistance of a conductor whether the conductor is ohmic or non-ohmic.

A conductor obeys Ohm's law if the graph b/w i and V for this conductor is a straight line i.e R is independent of V and r.

The macroscopic form of the relation $V = iR$ is $\vec{E} = \rho\vec{J}$.

A conducting material is said to obey Ohm's law if graph b/w E and J is a straight line i.e ρ is independent of E and J.

Ohm's law is a specific property of certain materials. It is not a general law of electromagnetism like Gauss's law.

6 Analogy between Current and Heat Flow

We can establish an analogy b/w current (flow of charge) and flow of heat.

Consider a thin electrically conducting slab of thickness Δx and area A. If a potential difference ΔV is applied b/w the opposite faces then the current flowing through the slab according to Ohm's law is given by

$$i = \frac{\Delta V}{R}$$

But $R = \frac{\rho L}{A}$

$R = \frac{\rho \Delta x}{A}$

$$i = \frac{\Delta V A}{\rho \Delta x}$$

If the thickness is very small i.e dx then the above relation becomes,

$$i = \frac{dV A}{\rho dx} \qquad \because \frac{1}{\rho} = \sigma$$

$$\therefore i = \sigma A \frac{dV}{dx}$$

$$i = -\sigma A \frac{dV}{dx}.$$

Where -ve sign shows that +ve charge moves in the direction of decreasing V i.e i is +ve when $\frac{dV}{dx}$ is -ve.

$$\because i = \frac{dq}{dt}.$$

$$\therefore \frac{dq}{dt} = -\sigma A \frac{dV}{dx} \quad -(A)$$

This equation shows the flow of charge. The analogus of equation A for heat flow is given by

$$\frac{dQ}{dt} = -KA \frac{dT}{dx} \quad -(B).$$

Where K is thermal conductivity which corresponds to electrical conductivity σ. The -ve sign stands for the loss of heat.

Comparing (A) and (B) we see that thermal conductivity K corresponds to electrical conductivity σ and temperature gradient $\frac{dT}{dx}$ corresponds to potential gradient $\frac{dV}{dx}$.

Moreover, heat and charge are carried by the free electrons in metals. A good electric conductor is also a good conductor of heat.

7. Microscopic View of Ohm's Law

Ohm's law is not a fundamental law of electromagnetism because it depends on the properties of conducting medium.

In a metal, the current is due to motion of free electrons called conduction electrons.

In the absence of electric field, these conduction electrons move

... making the motion in ... The electrons collide with ionic core of the lattice and suffers a change in direction of motion.

Now we define two terms.

(i) Mean free path λ (ii) Mean free time T.

λ is the distance b/w two consecutive collisions and T is the time b/w two consecutive collisions.

When we apply an electric field to a metal, the electrons change their random motion. Such that they drift with speed V_d opposite to electric field. The electron experiences a force $F = eE$.

So acceleration produces due to electric force is

$$a = \frac{F}{m} = \frac{eE}{m}.$$

$$\therefore a = \frac{eE}{m} \quad \text{---(i)}.$$

Consider an electron that has just collided with an ion core. So the electron has acquired a drift speed V_d given as,

$$V_d = aT$$

Putting the value of (a) from (i) we get,

$$V_d = \frac{eE}{m} T \quad \text{---(ii)}$$

In terms of current density V_d is given by,

$$J = neV_d.$$

$$V_d = \frac{J}{ne}. \quad \text{---(iii)}$$

Comparing (ii) and (iii) we get

$$\frac{J}{ne} = \frac{eE}{m} T.$$

By cross multiplication we get,

$$mJ = ne^2 E T.$$

$$\text{or} \quad \frac{m}{ne^2 T} = \frac{E}{J}.$$

$$\text{But} \quad \frac{E}{J} = \rho.$$

$$\therefore \rho = \frac{m}{ne^2 T} \quad \text{---(iv)}$$

In equation m, n, e are constants.

ow from this equation we can say that metals obey Ohm's law if τ is constant. This is only possible when τ does not depend on E and so ρ is independent of E. τ depends on the distribution of conduction electrons. This distribution is affected very slightly by very very large electric field. So value of τ remains unchanged when field is applied. So R.H.S of eq.(iv) is independent of E. Hence ρ is independent of E. So the metals obey Ohm's law.

Sample Problem 5

Sample Problem 5 (*a*) What is the mean free time τ between collisions for the conduction electrons in copper? (*b*) What is the mean free path λ for these collisions? Assume an effective speed of 1.6×10^6 m/s.

Sol:- $m = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $\rho = 1.69 \times 10^{-8}\ \Omega \cdot m$

$n = 8.49 \times 10^{28} / m^3$ (from Sample prob. 2), $\bar{V} = 1.6 \times 10^6$ m/s

(a) $\tau = ?$

As $$\tau = \frac{m}{ne^2\rho}$$

$$= \frac{9.11 \times 10^{-31}}{(8.49 \times 10^{28})(1.6 \times 10^{-19})^2(1.69 \times 10^{-8})}$$

$$= \frac{9.11}{36.73} \times 10^{-13} = 0.248 \times 10^{-13}$$

$$\boxed{\tau = 2.48 \times 10^{-14}\ \text{Sec}}$$ Ans.

(b) $\lambda = ?$

As $\lambda = \tau \bar{V}$

$= 2.48 \times 10^{-14} \times 1.6 \times 10^6$

$= 3.968 \times 10^{-8} = 4 \times 10^{-8}$ m.

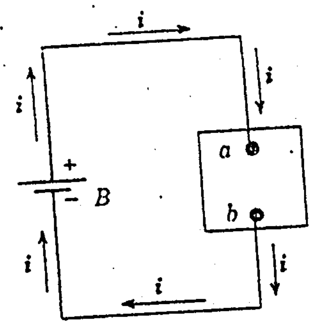
$\lambda = 40 \times 10^{-9}$ m $= \boxed{40\ nm}$ Ans

8. Energy Transfer in an Electric Circuit.

Consider a circuit c of a battery B connected to a black box.

"The black box is a box, whose contents are unknown".

A constant current i is flowing in the connecting wires and a constant potential difference V_{ab} exists b/w the terminals 'a' and 'b'. The black box may contain a resistor, a motor or a battery etc.



The potential energy of charge dq that flows through the box from a higher potential at (a) to a lower potential at (b) decreases by

According to law of conservation of energy this energy is transferred from electrical energy to some other form.

The other form of energy depends on the content of the box.

The energy transferred inside the box in time dt is given by

$$du = dq\,V_{ab} \qquad \because i = \frac{dq}{dt}, \quad dq = i\,dt.$$

$$\therefore du = i\,dt\,V_{ab}.$$

Now the power is defined as rate of transfer of energy

$$\therefore P = \frac{du}{dt}.$$

$$\therefore P = \frac{i\,dt\,V_{ab}}{dt}.$$

$$P = i V_{ab} \qquad V_{ab} = V.$$

$$\boxed{P = iV} \quad \text{— (1)}$$

If the box contains a motor, then this energy appears as mechanical energy.

If the box contains a battery; then energy appears as chemical energy of this third battery.

If the box contains a resistor, then this energy appears as internal energy due to which temperature of resistor increases.

The other forms of equation① are

$$\boxed{P = i^2 R} \quad — ②$$

$\because V = iR$.

$$\boxed{P = \frac{V^2}{R}} \quad — ③$$

Eq. ① deals with transfer of electrical energy to all other kinds. Equation ② and ③ deal with transfer of electrical energy to internal energy of the resistor.

The equations ② and ③ represent Joule's law and the energy dissipated is called Joule heating.

The S.I unit of power is Watt

There is another unit from the expression

$P = Vi$ which is volt Amp.

Watt and Volt ampere are indentical. It can be shown as follows.

$$1 \text{ Volt - Amp} = \frac{1 \text{ Joule}}{\text{coulomb}} \times \text{amp}.$$

$$= \frac{\text{Joule}}{\text{coul}} \times \frac{1 \text{ coul}}{\text{Sec}}.$$

$$= \frac{\text{Joule}}{\text{sec}}$$

$$\boxed{\therefore 1 \text{ Volt-amp} = \text{Watt}}.$$

Sample Problem-6

Sample Problem 6 You are given a length of heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. It is to be connected across a 120-V line. Under which circumstances will the wire dissipate more heat: (a) its entire length is connected across the line, or (b) the wire is cut in half and the two halves are connected in parallel across the line?

Sol. $R = 22\,\Omega$; $V = 120$ Volt.

(a) Power dissipated by the entire wire is

$$P = \frac{V^2}{R} = \frac{120 \times 120}{72}$$

$$\boxed{P = 200 \text{ Watt}} \quad \text{Ans}$$

(b). Power dissipated by half length of wire is

$$P' = \frac{V^2}{\frac{1}{2}R} = \frac{120 \times 120}{72 \times \frac{1}{2}} = \frac{120 \times 120}{36}.$$

As there are two halves

∴ Total power from both halves is = $\boxed{800 \text{ Watt}}$ Ans.

9. Semiconductors:

"A semiconductor material is that whose resistivity lies b/w those of conductors and insulators."

They are neither good conductors nor good insulators. These are the elements of 4^{th} group of the periodic table. It means they have four electrons in their outermost orbit.

The important examples are Si and Ge. These semi conductors are in the form of crystals. The important property of semi-conductors is that their conductivity can be changed by

(i) Increasing the temperature of semiconductor.

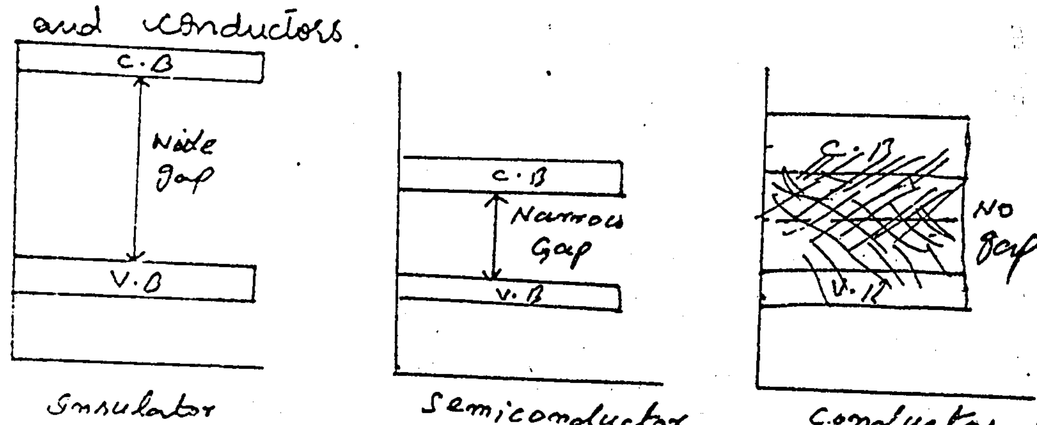
(ii) Applying voltage.

(iii) Falling light energy.

(iv) Doping the semiconductors with impurities.

With the help of band theory of solids, we can distinguish among conductors, semiconductors and insulators. So properties of conductors, semi conductors and insulators can be studied by band theory.

The following figures show energy bands in insulators, semi-conductors and conductors.



In an insulator the valence band is completely filled while conduction band is empty. The electrons in the valence band are tightly bound by the nucleus. Also there is a wide gap b/w valence band and conduction band. Thus the number of electrons which can be excited from the valence band to the conduction band is also zero. This explains the poor conductivity of insulators.

In Semi conductors, the gap b/w valence band and conduction band is small ≈ 1eV as compared to insulators. In semi-conductor, valence band is almost filled and conduction band is almost empty.

In conductors the valence band and conduction bands overlap and there is no gap b/w them.

Properties of Semiconductors:

(i) The resistivity of semiconductor is less than an insulator but more than a conductor.

(ii) Semiconductors have negative temperature coefficient of resistivity. i.e. the resistivity of semiconductors decreases with the increase in temperature and vice versa. e.g Ge is an insulator at low temperature but it becomes a good conductor at high temperatures.

(iii) When a suitable impurity is added to a semi conductor, its conductivity increases.

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10. Superconductors:

When we decrease the temperature of a conductor, its resistance decreases.

Let us see what happens, when we approach the absolute zero of the temperature.

According to Quantum mechanics the atoms of a conductor retain a minimum resistance due to lattice defects and impurities and due to minimum vibrational motion. Many materials show this type of behaviour.

But some materials show a different kind of behaviour at low temperatures.

In 1911 the Dutch physicist Kamerling onnes discovered that the resistance of Hg drops abruptly to zero at about 4K. He declared that Hg has passed into a new state in which its resistance has vanished. This state of Hg is called superconductive state and the phenomenon is called superconductivity. Electronic currents flow in the superconductors in the absence of potential difference and there is no heating effect due current in a superconductor.

The temperature at which a material becomes superconductor is called critical temperature T_c.

It is different for different materials when a superconductor is cooled below critical temperature, there is not only abrupt loss of resistivity but also changes in magnetic properties i.e at the critical temp, the resistance vanishes and at the same time the material shows diamagnetism.

The properties of the material are different above and below the critical temperature. e.g

Properties of Superconductors:

(i) At the critical temperature T_c the d.c resistance vanishes and below T_c it remains zero.

(ii) At the critical temperature, the specific heat of materials increases rapidly. Below T_c the variation of specific heat is different from that above T_c.

So at the critical temperature there is a change from one set of properties to matter.

So we say that below T_c, the material is in the superconductive