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Chapter - 35 1

AMPERE'S LAW

1 Biot-Savart Law

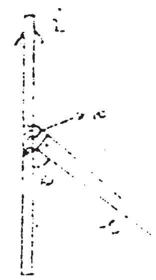
This law enables us to find the flux density $\vec{B}$ due to certain current carrying conductors.

Suppose a current i is flowing upward in a straight wire. We know that a magnetic field is produced around the wire.

Suppose we want to find the magnetic field strength $\vec{B}$ at any point P the wire.

Let us take an element of length AC = ds (say). Draw ⊥ AC to CP.

As AB is small



$$\therefore \angle\theta \approx \angle ACB$$

$$\therefore \frac{AB}{AC} = \sin\theta$$

$$AB = AC\sin\theta$$

$$\boxed{\therefore AB = ds\sin\theta}$$

According to Laplace, it is found that the magnetic induction dB at pt. P due to element ds of conductor carrying current i depends on different factors as below

(i) $dB \propto i$

(ii) $dB \propto ds\sin\theta$

(iii) $dB \propto \frac{1}{r^2}$

When $ds\sin\theta$ is the effective length of the element as seen from pt. P.
r is the distance of point P from the element.

Combining the above results we get,

$$dB \propto \frac{i\,ds\sin\theta}{r^2}$$

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$$dB = \frac{K i\, ds \sin\theta}{r^2}.$$

Here 'K' is a constant whose value is $K = \frac{\mu_0}{4\pi}$.
where $\mu_0 = 4\pi \times 10^{-7}$ weber/Amp-m. is called the permeability of free space.

$$\therefore dB = \frac{\mu_0 i\, ds \sin\theta}{4\pi r^2}.$$

In vector form it is written as,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2} \hat{n} \quad \text{—— (1).}$$

Where $\hat{n}$ is a unit vector showing the direction of field and is determined by right hand rule.
If $\hat{i}$ is a unit Vector along a current element and $\hat{r}$ is a unit vector along position vector $\vec{r}$ of point P. Then

$$\hat{i} \times \hat{r} = 1 \times 1 \times \sin\theta\, \hat{n}.$$

$$\hat{i} \times \hat{r} = \sin\theta\, \hat{n}.$$

$\therefore$ Expression (1) becomes

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\, ds\, (\hat{i} \times \hat{r})}{r^2}$$

$$\therefore \vec{dB} = \frac{\mu_0}{4\pi} \frac{i\, \vec{ds} \times \hat{r}}{r^2} \qquad \text{But } \vec{ds} = ds\,\hat{i}.$$

$$\boxed{\therefore \vec{dB} = \frac{\mu_0}{4\pi} \frac{i\, \vec{ds} \times \vec{r}}{r^3}} \qquad \text{But } \hat{r} = \frac{\vec{r}}{r}.$$

The magnetic induction B at point 'P' due to the whole wire is given by integrating,

$$\int \vec{dB} = \frac{\mu_0 i}{4\pi} \int \frac{\vec{ds} \times \vec{r}}{r^3}.$$

$$\vec{B} = \frac{\mu_0 i}{4\pi} \int \frac{\vec{ds} \times \vec{r}}{r^3}.$$

This is Biot - Savart's Law or Ampere - Biot Savarat's law.

Applications

) Field due to Current in a Straight Conductor

Consider a long straight wire carrying current i extending from $-\infty$ to ∞.
We want to find the field strength B due to it at a point P whose ⊥ distance from wire is R.
The magnetic induction due to an element of length dS is given by Biot Savart's law as

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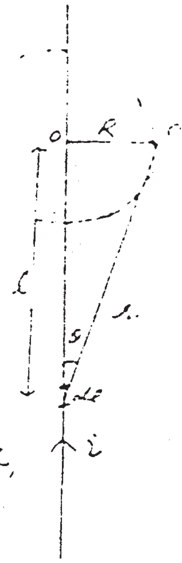
$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2} \quad \text{—— (1)}$$

Suppose the element is taken at a distance x below the point O which is taken as the origin. Then as shown,



$$-\frac{x}{R} = \cot\theta$$

$$-x = R\cot\theta$$

$$\text{or} \quad -dS = R(-\operatorname{cosec}^2\theta\, d\theta) \quad \text{—— (i)}$$

$$\text{or} \quad ds = R\operatorname{cosec}^2\theta\, d\theta$$

Also
$$\frac{r}{R} = \operatorname{cosec}\theta$$

$$r = R\operatorname{cosec}\theta \quad \text{—— (ii)}$$

Putting the values of ds and r from (i) and (ii) in (1) we get,

$$dB = \frac{\mu_0}{4\pi} \frac{i\,(R\operatorname{cosec}^2\theta\, d\theta)\sin\theta}{R^2\operatorname{cosec}^2\theta}$$

$$dB = \frac{\mu_0 i \sin\theta\, d\theta}{4\pi R}$$

The field at P due to the whole wire is obtained by integrating the above expression

$$B = \frac{\mu_0 i}{4\pi R}\int_{-\infty}^{+\infty} \sin\theta\, d\theta$$

$\because$ when $x \to -\infty$, $\theta = 0$ and when $x \to \infty$, $\theta = \pi$

$$= \frac{\mu_0 i}{4\pi R}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{\mu_0 i}{4\pi R}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{\mu_0 i}{4\pi R}(1+1) = \frac{\mu_0 i}{4\pi R}\times 2$$

$$B = \frac{\mu_0 i}{2\pi R}$$

In vector form

$$\vec{B} = \frac{\mu_0 i}{2\pi R}\hat{k}$$ where $\hat{k}$ is a unit vector which represents the direction of the tangent to the circle at P

Magnetic Field due to a Circular Current Loop

Consider a circular loop of radius R carrying current i. We want to find flux density B at a point P on the axis of loop at a distance Z from the centre of the loop.

Take a small element of length ds of the loop as shown

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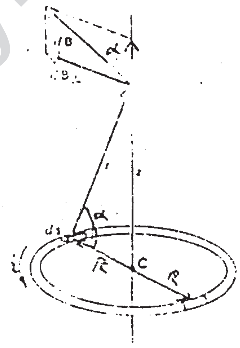
he angle θ b/w $\vec{ds}$ and $\vec{r}$ is 90°.
om Biot_Savart's law we find that
agnetic induction $\vec{dB}$ is $\perp$ to $\vec{r}$
nd $\vec{ds}$.
putting $\theta = 90^\circ$ in the equation of Biot Savart's law we get,

$$dB = \frac{\mu_0 i}{4\pi} \frac{ds \sin\theta}{r^2}.$$

$$\therefore\ dB = \frac{\mu_0 i}{4\pi} \frac{ds}{r^2} \quad \text{—— (1).} \qquad \sin 90^\circ = 1.$$

; us resolve $\vec{dB}$ into two rectangular components $dB_\perp$ and $dB_{\parallel}$.
. is $\perp$ to the axis and $dB_{\parallel}$ is parallel to the axis of loop.
o from symmetry, we find that $dB_\perp$ has no contribution to the
d at pt. P because the $\perp$ components due to all elements of the
. cancel away each other. Only the components which a'
allel to the axis are added up.
nce total field B due to the entire loop is

$$B = \int dB_{\parallel} \quad \text{—— (A).}$$



From the

$dB_{\parallel} = dB \cos\alpha$.

ting the value of dB from (1) we get

$$dB_{\parallel} = \frac{\mu_0 i}{4\pi} \frac{dS}{r^2} \cos\alpha.$$

$$dB_{\parallel} = \frac{\mu_0 i\, dS}{4\pi r^2} \cos\alpha \quad \text{—— (2).}$$

ow from fig

$$r^2 = (R^2 + Z^2)$$

$$r = \sqrt{R^2 + Z^2}$$

$$\text{and} \quad \cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + Z^2}}.$$

Putting the values of r^2 and $\cos\alpha$ in (2) we get

$$dB_{\parallel} = \frac{\mu_0 i . dS}{4\pi (R^2 + Z^2)} \frac{R}{\sqrt{R^2 + Z^2}}.$$

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi} \frac{dS}{(R^2 + Z^2)^{3/2}}.$$

Putting $dB_{\parallel}$ in (A) we get,

$$B = \int dB_{\parallel} \quad \text{—— (A).}$$

$$B \int \frac{\mu_0 i R}{4\pi} \frac{dS}{(R^2 + Z^2)^{3/2}}.$$

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$$B = \frac{\mu_0 i R}{4\pi (R^2+Z^2)^{3/2}} \int dS$$

$$B = \frac{\mu_0 i R}{4\pi (R^2+Z^2)^{3/2}} \, 2\pi R \qquad \int dS = 2\pi R$$

$$\boxed{B = \frac{\mu_0 i R^2}{2 (R^2+Z^2)^{3/2}}} \qquad -(3).$$

This is the magnetic induction on the axis of loop.

At the centre of loop $Z = 0$.

$\therefore$ Magnetic flux density at the centre of loop is

$$B = \frac{\mu_0 i R^2}{2 R^3}$$

$$\boxed{B = \frac{\mu_0 i}{2R}}$$

The magnetic induction B is maximum at the centre of loop and decreases with increase in Z.

If $Z \gg R$ then eq.(3) becomes

$$\boxed{B = \frac{\mu_0 i R^2}{2 Z^3}}$$

This is the expression for magnetic field for loop having single turn. For a loop consisting of N turns, this field becomes,

$$B = \frac{\mu_0 N i R^2}{2 Z^3}.$$

multiplying and dividing by π we get,

$$B = \frac{\mu_0 N i \pi R^2}{2\pi Z^3}.$$

But πR^2 = Area of loop = A.

$$\therefore \quad B = \frac{\mu_0 N i A}{2\pi Z^3}.$$

But $NiA = \mu$ = The magnetic dipole moment of the current loop.

$$\boxed{\therefore \; B = \frac{\mu_0 \mu}{2\pi Z^3}}$$

3. Force b/w Two Long Parallel Current Carrying Conductors.

Consider two straight wires A and B each of length L placed || to each other. r distance apart carrying currents i_1 and i_2 in the

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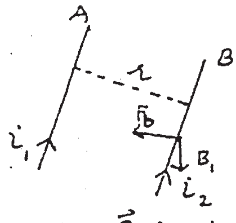
same direction respectively.
The current passing through each wire produces a magnetic field around it and each wire is placed in the magnetic field produced by the other.
The magnetic field due to current i_1 through wire A at wire B is given by



$$B_a = \frac{\mu_0 i}{2\pi r} \quad \text{—— (1).}$$

The direction of B_1 is given by right hand rule and is into the plane of page.
The wire B placed in the magnetic field B_1 experiences a force $\vec{F_2}$ given by

$$\vec{F_b} = i_2 (\vec{L} \times \vec{B_1}).$$

The force F_b is directed towards wire A. The direction of wire B is $\perp$ to B. ∴ Magnitude of $\vec{F_b}$ is given by

$$F_b = i_2 L B_1 \sin 90°.$$

$$F_b = i_2 L B_1.$$

Putting the value of B_1 from (1) we get

$$F_b = i_2 L \left(\frac{\mu_0 i}{2\pi r} \right).$$

$$\boxed{F_b = \frac{\mu_0 L i_1 i_2}{2\pi r}}$$

Similarly force on wire A carrying current i_1 due to magnetic field of current i_2 is given by.

$$\vec{F_a} = i_1 (\vec{L} \times \vec{B_2}).$$

$$F_a = i_1 L B_2.$$

$$F_a = \frac{i_1 \mu_0 L i_1 i_2}{2\pi r}.$$

$$\boxed{F_a = \frac{\mu_0 L i_1 i_2}{2\pi r}}$$

This force $\vec{F_a}$ is directed towards wire B. The two forces are equal and opposite. So the wires attract each other.
If the direction of current in one wire is opposite to that in the other, the wires repel each other.

So we find that

$$F_a = F_b = F = \frac{\mu_0 i_1 i_2 L}{2\pi r} \quad \text{—— (2).}$$

From eq. (2) we can get the definition of Ampere as follows
"The current flowing in each of the two parallel wires having unit length placed one meter apart giving rise to a force of 2×10^{-7} N."

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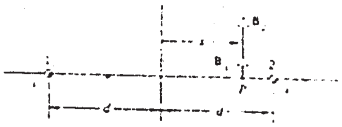
Sample Problem - 1

Sample Problem 1 Two long parallel wires a distance $2d$ apart carry equal currents i in opposite directions, as shown in Fig. 7a. Derive an expression for the magnetic field B at a point P on the line connecting the wires and a distance x from the point mid-way between them.

Sol:- The currents are in opposite direction.
The magnetic field B_1 due to i_2 are in same direction at P.

$$B_1 = \frac{\mu_0 i}{2\pi(d+x)} \quad ; \quad B_2 = \frac{\mu_0 i}{2\pi(d-x)}$$

$$B = B_1 + B_2.$$



$$= \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0 i}{2\pi(d-x)}$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{d+x} + \frac{1}{d-x}\right].$$

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{d-x+d+x}{d^2-x^2}\right)$$

$$= \frac{\mu_0 i}{2\pi}\left(\frac{2d}{d^2-x^2}\right)$$

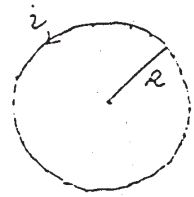
$$\boxed{B = \frac{\mu_0 i d}{\pi(d^2-x^2)}} \quad \text{Ans.}$$

Sample Problem - 2

Sample Problem 2 In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.29×10^{-11} m at a frequency ν of 6.63×10^{15} Hz (or rev/s). (a) What value of B is set up at the center of the orbit? (b) What is the equivalent magnetic dipole moment?

Sol:- Radius of orbit $= R = 5.29 \times 10^{-11}$ m.
Frequency $= \nu = \frac{1}{T} = 6.63 \times 10^{15}$ Hz. $\mu_0 = 4\pi \times 10^{-7}$ wb/Amp.

(a) B at the centre of orbit = ?
As circulating electron is equivalent to a current loop.
$\therefore$ B at the centre is given by,



$$B = \frac{\mu_0 i}{2R}$$

$$i = \frac{e}{T} = e\nu.$$

$$\therefore B = \frac{\mu_0}{2R} e\nu.$$

$$= \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 6.63 \times 10^{15}}{2 \times 5.29 \times 10^{-11}}.$$

$$= \frac{4 \times 3.14 \times 1.6 \times 6.63}{2 \times 5.29} \times 10^{-26+26}.$$

$$= 12.59$$

$$\boxed{B = 12.6\ T} \quad \text{Ans.}$$

(b). Magnetic dipole moment $= NiA$. Here $N = 1$.

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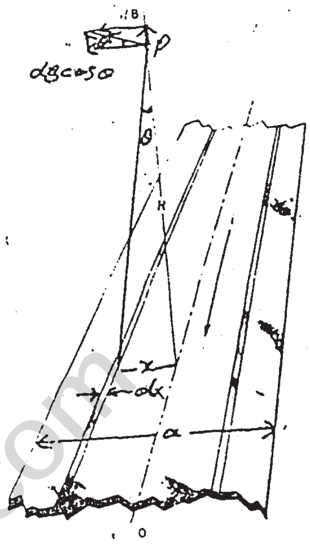
$\therefore \quad \mu = iA.$

$= e\nu A = e\nu \times \pi R^2.$

$= 1.6\times10^{-19}\times6.63\times10^{5}\times3.14\times(5.29\times10^{-11})^2.$

$= 1.6\times6.63\times3.14\times5.29\times5.29\times10^{-26}.$

$\underline{\mu = 9.32\times10^{-24}\ A\text{-}m^2}$ Ans.



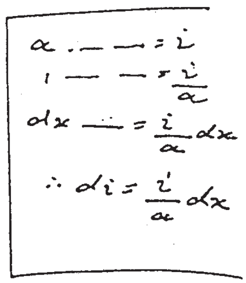
Sample Problem 3

Sample Problem 3 Figure 8 shows a flat strip of copper of width a and negligible thickness carrying a current i. Find the magnetic field B at point P, at a distance R from the center of the strip along its perpendicular bisector

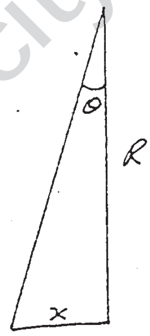
Sol. B = ? at point P. width of whole strip = a.
Let us divide the strip into long small elements of width dx each of which acts as a wire carrying current di.
The magnetic field dB due to element is given by

$$dB = \frac{\mu_0 di}{2\pi r}.$$

$$= \frac{\mu_0}{2\pi r}\frac{i}{a}dx.$$



$$\boxed{dB = \frac{\mu_0 i\, dx}{2\pi r a}} \quad \text{—(1).}$$



From the fig. $\frac{R}{r} = \cos\theta.$

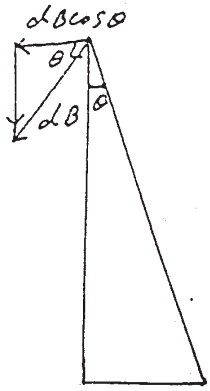
$\therefore \frac{r}{R} = \sec\theta.$

$r = R\sec\theta$ —(i)

$\therefore$ Putting the value of r in (1) we get.

$$dB = \frac{\mu_0 i dx}{2\pi R\sec\theta . a} \quad (2).$$

Now the vertical components have no contribution to $\vec{B}$. Only horizontal components $dB\cos\theta$ are added up.



$\therefore$ Total field at P is.

$$B = \int dB\cos\theta.$$

Putting the value of dB from (2) we get,

$$B = \frac{\mu_0 i}{2\pi aR}\int\frac{dx\cos\theta}{\sec\theta}.$$

$$= \frac{\mu_0 i}{2\pi aR}\int\frac{dx}{\sec^2\theta}.$$

From fig. $\frac{x}{R} = \tan\theta, \quad x = R\tan\theta.$

$dx = R\sec^2\theta\, d\theta.$

Putting the value of dx in (3) we get

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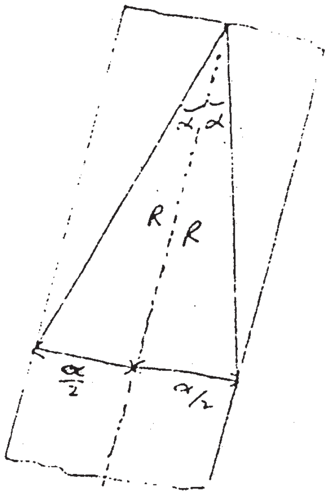
$$B = \frac{\mu_0 i R}{2\pi a R} \int \frac{\sec^2\theta \, d\theta}{\sec^2\theta}$$

$$B = \frac{\mu_0 i}{2\pi a} \int d\theta$$



From figure

$$\frac{a/2}{R} = \tan\alpha$$

$$\therefore \frac{a}{2R} = \tan\alpha$$

$$\alpha = \tan^{-1}\left(\frac{a}{2R}\right)$$

Putting the limits on $\theta = \pm\alpha$ we get.

$$B = \frac{\mu_0 i}{2\pi a} \int_{-\alpha}^{+\alpha} d\theta$$

$$= \frac{\mu_0 i}{2\pi a} \left|\theta\right|_{-\alpha}^{+\alpha} = \frac{\mu_0 i}{2\pi a} (\alpha + \alpha)$$

$$= \frac{2\mu_0 i \alpha}{2\pi a} = \frac{\mu_0 i \alpha}{\pi a}$$

Putting $\alpha = \tan^{-1}\dfrac{a}{2R}$

$$\boxed{B = \frac{\mu_0 i}{\pi a} \tan^{-1}\frac{a}{2R}}$$

This is general expression for B due to strip.
For points very far from the strip $\alpha \simeq \tan\alpha$.

$$\therefore B = \frac{\mu_0 i}{\pi a}\left(\frac{a}{2R}\right)$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$ Ans.

Sample Problem-4

Sample Problem 4 A long horizontal rigidly supported wire carries a current i_a of 96 A. Directly above it and parallel to it is a fine wire that carries a current i_b of 23 A and weighs 0.073 N/m. How far above the lower wire should this second wire be strung if we hope to support it by magnetic repulsion?

Sol. $i_a = 96\,A$; $i_b = 23\,A$; $\mu_0 = 4\pi\times10^{-7}\ wb/A\text{-}m$. Weight per unit length: .073 N/m.

For equilibrium

Force per unit length = Wt. per unit length.

$$\frac{F}{L} = .073 \quad \text{— (1)}$$

$$\because F = \frac{\mu_0 i_a i_b L}{2\pi d}$$

$$\therefore \frac{\mu_0 i_a i_b L}{L\, 2\pi d} = 0.073$$

$$d = \frac{\mu_0 i_a i_b}{2\pi \times .073} = \frac{4\pi\times10^{-7}\times 96\times 23}{2\pi \times .073}$$

$$= 6.0493\times10^{-3}$$

$$\boxed{d = 6.0\ mm}$$ Ans.

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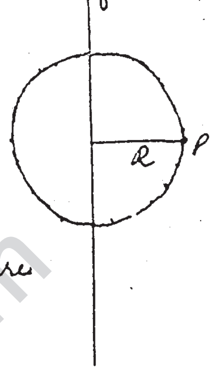
4- Ampere's Circutal Law:

(a) Integral Form:

Consider a straight wire in which current is flowing. We know that a magnetic field is produced around it. We want to determine the field strength due to this wire at any point P distant R from it.

We know that this field is given by,

$$\vec{B} = \frac{\mu_0 i}{2\pi R}\hat{k} \quad \text{—— (1).}$$



The symmetry shows that the magnetic induction $\vec{B}$ is same everywhere on the circumference of circle passing through point P and having its centre at the wire.

The above relation can also be expressed as,

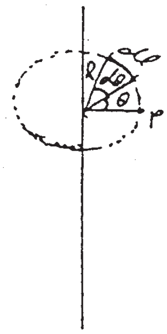
$$B \times 2\pi R = \mu_0 i \quad \text{—— (2).}$$

It shows that product of magnetic field strength and length of path parallel to the field is equal to μ_0 times the current enclosed. This is called Ampere's Law. It holds for a path of any shape.

For more general case, eq.(1) can be written as

$$\oint \vec{B}.\vec{dl} = \oint \frac{\mu_0 i}{2\pi R}\hat{k}.\vec{dl}$$

$$= \frac{\mu_0 i}{2\pi R}\int \hat{k}.\vec{dl}$$



$$= \frac{\mu_0 i}{2\pi R}\oint dl$$

Now $dl = Rd\theta$.

$$\oint \vec{B}.\vec{dl} = \frac{\mu_0 i}{2\pi R} R\oint d\theta$$

$$= \frac{\mu_0 i R}{2\pi R}\left|\theta\right|_0^{2\pi} = \frac{\mu_0 i}{2\pi}\times 2\pi$$

$$\oint \vec{B}.\vec{dl} = \mu_0 i.$$

This is called Ampere's Circuital law

Now $\quad i = \int \vec{J}.\vec{ds}$

$$\boxed{\therefore \oint \vec{B}.\vec{dl} = \mu_0\int \vec{J}.\vec{ds}.}$$

This gives integral form of Ampere's law.

According to it line integral of magnetic induction $\vec{B}$ due to current i around any closed path is μ_0 times the current enclosed.

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(b) Differential Form:

According to integral form of Ampere's law

$$\oint \vec{B}\cdot\vec{ds} = \mu_0 \int_S \vec{J}\cdot\vec{ds} \quad \text{— (1).}$$

Using Stokes theorem on L.H.S. we get

$$\oint \vec{B}\cdot\vec{ds} = \int_S \text{curl}\,\vec{B}\cdot\vec{ds}$$

$\therefore$ Expression (1) becomes

$$\int_S \text{curl}\,\vec{B}\cdot\vec{ds} = \mu_0 \int_S \vec{J}\cdot\vec{ds}.$$

or

$$\int_S (\text{curl}\,\vec{B} - \mu_0 \vec{J})\cdot\vec{ds} = 0$$

$$\text{curl}\,\vec{B} - \mu_0 \vec{J} = 0$$

$$\boxed{\therefore\ \text{Curl}\,\vec{B} = \mu_0 \vec{J}.}$$

This is called Differential form of Ampere's Law.

Sample Problem - 5

Sample Problem 5 Derive an expression for B at a distance r from the center of a long cylindrical wire of radius R, where r < R. The wire carries a current i, distributed uniformly over the cross section of the wire.

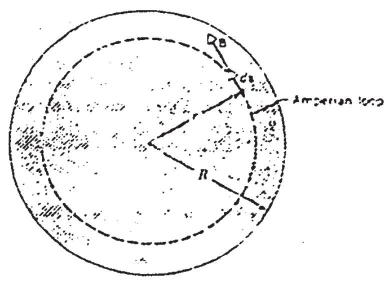
Sol: Draw an Amperian loop of radius r < R.
Symmetry shows that $\vec{B}$ is uniform along the loop and tangent to it.
$\therefore$ Using Ampere's law we get

$$B(2\pi r) = \mu_0 \times \text{current enclosed by Amperian loop.}$$

$$\therefore B(2\pi r) = \mu_0 \times i\,\frac{\pi r^2}{\pi R^2}$$

$$B = \frac{\mu_0 i \pi r^2}{\pi R^2 \times 2\pi r}$$

$$\boxed{B = \frac{\mu_0 i r}{2\pi R^2}} \quad \text{Ans.}$$



Applications of Ampere's Law

(a) Magnetic Field due to a Solenoid

A solenoid is a cylinderical frame tightly wound by an insulated wire. i.e it is a long wire wound in a closed packed helix. If we examine the field of a solenoid by drawing the induction lines, we find that the field due to a solenoid is almost all within it and parallel to its length. The field outside the solenoid is negligible as compared to the field inside it. The solenoid field is the vector sum

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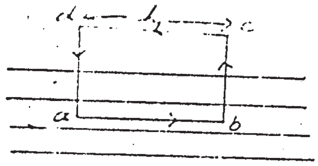
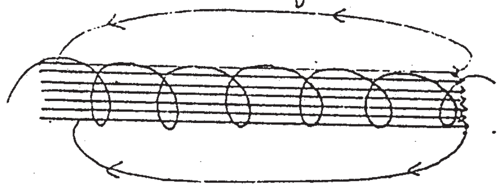
of the fields set up by all the turns. Fig. shows a section of solenoid.





Let us take a loop abcda of the solenoid and apply Ampere's law to it. The symmetry shows that the closed path can be divided into four elements $a \to b$, $b \to c$, $c \to d$ and $d \to a$. By ampere's law

$$\oint \vec{B} \cdot \vec{ds} = \mu_o \times \text{current enclosed.}$$

So the integral $\oint \vec{B} \cdot \vec{ds}$ can be written as the sum of four integrals.

$$\therefore \int_a^b \vec{B} \cdot \vec{ds} + \int_b^c \vec{B} \cdot \vec{ds} + \int_c^d \vec{B} \cdot \vec{ds} + \int_d^a \vec{B} \cdot \vec{ds} = \mu_o \times \text{Current enclosed}$$

We find that

$$\int_b^c B \cdot ds = \int_d^a \vec{B} \cdot \vec{ds} = 0 \qquad \because \theta = 90^\circ$$

Also $\int_c^d \vec{B} \cdot \vec{ds} = 0$ as the field outside the solenoid is neglegible.

$\therefore$ We are left with

$$\int_a^b \vec{B} \cdot \vec{ds} = \mu_o \times \text{current enclosed.}$$

Since B is constant from $a \to b$. So

$$B \int_a^b ds = \mu_o \times \text{current enclosed.}$$

$$\therefore B \times h = \mu_o \times \text{"}$$

where $h = ab$.

To find the current enclosed let n be the number of turns per unit length of the solenoid. So number of turns in length $h = nh$. If i_o is the current in the solenoid, Then

$$\text{current enclosed} = nh i_o.$$

$$\therefore Bh = \mu_o \times nh i_o.$$

$$\boxed{\therefore B = \mu_o n i_o.}$$

This is the field due to a solenoid. This equation shows that magnetic field inside a solenoid depends only on current i_o and the number of turns per unit length n.

A solenoid is a pratical way to obt[illegible] a uniform magnetic field.

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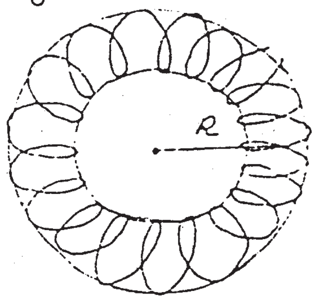
Field due to a Toroid:

A Toroid is a circular solenoid. Fig. shows a toroid of radius R. The symmetry shows that the field due to the toroid is everywhere constant with in the toroid and also parallel to its length.



Applying Ampere's circuital law we get

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times \text{current enclosed.}$$

$$\oint B\,dl \cos 0^\circ = \mu_0 \times \quad '' \quad ''$$

$$B \oint dl = \mu_0 \times \quad '' \quad ''$$

Now $\oint dl$ = Length of the toroid $= 2\pi R$

$$\therefore \quad B \times 2\pi R = \mu_0 \times \text{current enclosed.}$$

To find the current enclosed, let N be the total number of turns of the toroid. If i_0 is the current in the toroid, then current enclosed $= N i_0$

Therefore

$$B \times 2\pi R = \mu_0 N i_0$$

$$\therefore \quad B = \frac{\mu_0 N i_0}{2\pi R}$$

This is the field due to toroid.

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Sample Problem-6

Sample Problem 6 A solenoid has a length $L = 1.23$ m and an inner diameter $d = 3.55$ cm. It has five layers of windings of 850 turns each and carries a current $i_0 = 5.57$ A. What is B at its center?

Solution. $L = 1.23\,m$, $\mu_0 = 4\pi \times 10^{-7}$ wb/A-m, dia $d = 3.55$ cm

$N = 850$, $i_0 = 5.57$, $B = ?$

As for a Solenoid, we have

$$B = \mu_0 n i_0$$

$$= \frac{\mu_0 N i_0}{L} \qquad \because n = \frac{N}{L}$$

$$= \frac{4\pi \times 10^{-7} \times 5 \times 850 \times 5.57}{1.23}$$

$$= 241728 \times 10^{-7}$$

$$= 2.417 \times 10^{-2} = 2.42 \times 10^{-2}$$

$$= 24.2 \times 10^{-3}\,T$$

$$\boxed{B = 24.2\ mT} \quad \text{Ans.}$$

THE END.