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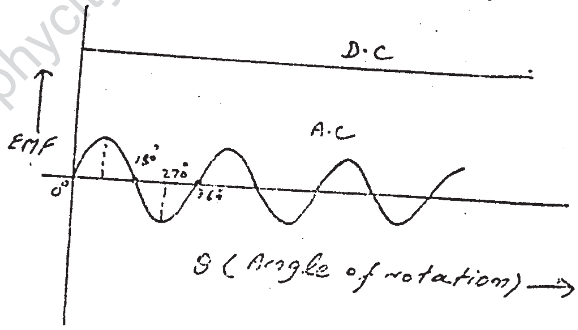
Chapter - 39

ALTERNATING CURRENT CIRCUITS

1- Alternating Current:

"A current which alternates (changes) or reverses its direction reversal time per second is called Alternating current." and is written as A.C

The alternating Emf can be produced by A.C generator. In an A.C. generator a coil is rotated b/w the poles of a magnet and an alternating emf is produced. This emf remains +ve during $\frac{1}{2}$ cycle and become -ve during other $\frac{1}{2}$ cycle. This cycle is divided into 360°. This emf rises and falls like a wave as shown.



The number of cycles completed per second is called frequency of A.C. The frequency of the A.C used in Pakistan is 50 cycles per sec. In U.S.A the frequency of A.C is 60 C.P.S. The source of A.C emf is represented by the symbol -⊖-. The alternating Emf at any time 't' is called instantaneous EMF and is given by,

$$E = E_m \sin\omega t.$$

where E = Instantaneous value of A.C. voltage.

E_m = Amplitude (Max. value) of A.C. Voltage.

$\omega = 2\pi\nu$ = Angular velocity (frequency) of coil

and ν is linear frequency.

The current in the circuit at any time 't' is

$$i = i_m \sin(\omega t - \varphi)$$

where i_m = Max. value of current

and φ = Phase angle or phase constant.

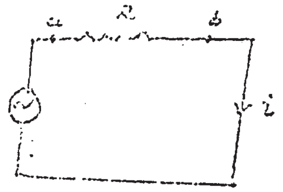
A.C. Current in Resistive Element:

Consider a resistance R connected in series with a source of A.C EMF as shown.

The potential difference across R is given by

$$E = V_R = V_a - V_b = iR$$

The current i in the circuit is



$$i = i_m \sin(\omega t - \varphi) \quad (1)$$

$$\therefore \quad V_R = i_m \sin(\omega t - \varphi) R$$

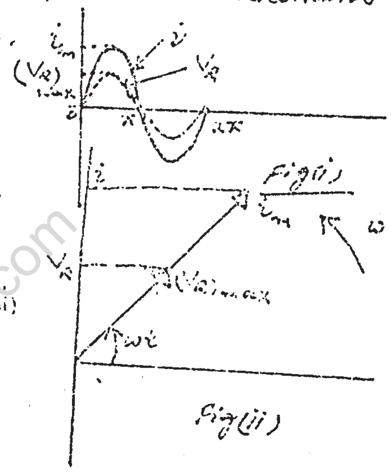
$$V_R = i_m R \sin(\omega t - \varphi) \quad (2)$$

From (1) and (2) we find that i and V_R are in phase which means that both voltage and current attains their maximum and minimum values at the same time.

Figs (i) shows the phase relationship.

The problem of AC network become very simple if we consider voltage and currents as phasor quantities.

Fig.(ii) shows the phasor diagram. In fig.(ii) phasors are represented by open arrows which rotate anticlockwise with angular frequency ω about the origin.



As in resistive element V_R and i are in phase. So V_R and i lie along the same line.

The phasors have the following properties.

i. The length of phasor is propotional to the maximum value of alternating quantity.

(ii) The projection of phasor on vertical axis gives the instantaneous value of the alternating quantity. The arrows on the vertical axis represent time varying quantities V_R and i.

As the phasor rotates, their projection on the vertical axis give sinusoidally varying current or voltage.

Power Dissipated in a Resistor:

The instantaneous power dissipated across the resistor R is given by

$$P = i^2 R$$

$$P = i_m^2 R \sin^2(\omega t - \varphi) \quad \because i = i_m \sin(\omega t - \varphi)$$

energy dissipated in the resistor fluctuates with time.
So we define average power dissipated during a particular cycle as $\bar{P} = \langle P \rangle = i_m^2 R \langle \sin^2(\omega t - \varphi) \rangle$.

$\because \langle \sin^2(\omega t - \varphi) \rangle = \frac{1}{2}$.

$$\therefore \langle P \rangle = \frac{1}{2} i_m^2 R.$$

$$\langle P \rangle = \left(\frac{i_m}{\sqrt{2}} \right)^2 R$$

But $\frac{i_m}{\sqrt{2}} = i_{rms}$, the root mean square value of current.

$$\boxed{\therefore \langle P \rangle = i_{rms}^2 R.}$$

As $\quad i_{rms} R = E_{rms}$.

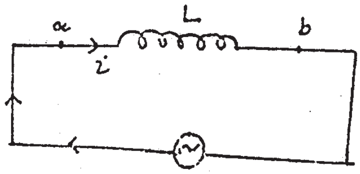
$$\therefore \langle P \rangle = (i_{rms} R)(i_{rms})$$

$$\boxed{\langle P \rangle = E_{rms} \times i_{rms}.}$$

It should be noted that resistance in electric circuits behaves as friction in mechanics. When a body is moved against friction, mechanical energy is dissipated in the form of heat.
Similarly, when current is passed through a resistance, electrical energy is converted into heat.

2- A.C. in an inductive element:

Consider a source of A.C emf connected to an inductor L as shown. The potential difference across the inductor is given by



$$V_L = V_a - V_b = L \frac{di}{dt}.$$

As $\quad i = i_m \sin(\omega t - \varphi) \quad (1)$

$$\therefore V_L = L \frac{d}{dt} i_m \sin(\omega t - \varphi)$$

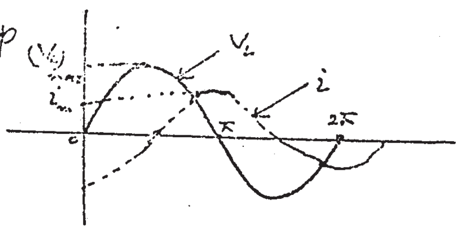
$$\boxed{V_L = L i_m \omega \cos(\omega t - \varphi)} \quad (2).$$

$\because \sin(\theta + \pi/2) = \cos\theta$.

$\therefore$ Expression (2) becomes

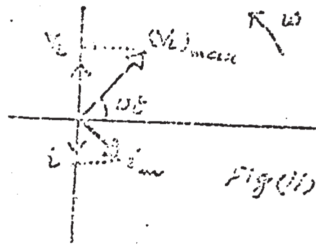
$$\boxed{V_L = L i_m \omega \sin(\omega t - \varphi)} \quad (3)$$

From (1) and (3) we find that i and V_L are not in phase. Here i and V_L are out of phase by $90°$ with V_L leads i by $\frac{\pi}{2}$ or current i lags behind the V_L by $90°$.
Fig (i) shows the phase relationship



Fig(i)

V_L and i are one quarter cycle out of phase.
Fig. (iii) shows the phasor diagram.
As the phasor rotates, it is clear that the phasor i lags behind the phasor V_L by one quarter cycle.



Fig(iii)

Inductive Reactance (X_L):

The term $\omega L = X_L$ is called inductive reactance. It is the resistance offered by the inductance. So it is measured in ohms.

$\therefore$ Expression (3) becomes

$$V_L = i_m X_L \sin(\omega t - \varphi + \pi/2) \qquad (4)$$

The maximum value of V_L is given by

$$(V_L)_{max} = i_m X_L \qquad \because (\sin\theta)_{max} = 1.$$

Where $X_L = \dfrac{i_m}{(V_L)_{max}}$ = Resistance or opposition offered by an inductor to the flow of current and is called reactance or inductive reactance.

Power Dissipated in an Inductor

The instantaneous power dissipated in an inductor is given by

$$P = V_L\, i$$

$$\therefore \quad P = V_L\, i_m \sin(\omega t - \varphi) \qquad \text{As } i = i_m \sin(\omega t - \varphi)$$

From eq. (2)

$$V_L = L\, i_m \cos(\omega t - \varphi)$$

$$\therefore \quad P = L\, i_m \omega \cos(\omega t - \varphi)\, i_m \sin(\omega t - \varphi)$$

$$\boxed{P = i_m^2\, \omega L \cos(\omega t - \varphi) \sin(\omega t - \varphi).}$$

Average Power Dissipated over a complete cycle:

Average power over a complete cycle is given by

$$\langle P \rangle = i_m^2\, \omega L \langle \cos(\omega t - \varphi) \rangle \langle \sin(\omega t - \varphi) \rangle.$$

$$\because \quad \langle \cos(\omega t - \varphi) \rangle = 0 \text{ and } \langle \sin(\omega t - \varphi) \rangle = 0$$

$$\therefore \quad \langle P \rangle = i_m^2\, \omega L\, (0)$$

$$\boxed{\langle P \rangle = 0}.$$

Hence average power dissipated in an inductive circuit over one cycle is zero. It means that if it absorbs some energy during a part of a cycle it will deliver an equal amount of energy during the rest of the cycle. Hence it can be used (without wastage of energy) to block A. C currents.

Energy stored in an Inductor:

As $P = \frac{dw}{dt} = V_L i = L \frac{di}{dt} i$

$$\frac{dw}{dt} = L \frac{di}{dt} i.$$

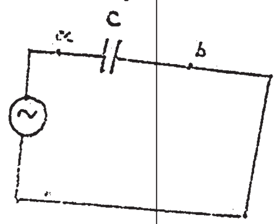
$$dw = Li\,di.$$

Integration gives
$$\int dw = L \int i\,di.$$
$$w = \frac{Li^2}{2}$$
$$w = \frac{1}{2} Li^2$$

The maximum energy stored is
$$E = W_{max}$$
$$\boxed{E = \frac{1}{2} L i_{max}^2}$$

3. A.C. Current in a Capacitive Element

Consider a source of alternating Emf applied across a capacitor having capacitance 'C'. Suppose i is the current through the circuit. Let the charge stored in the capacitor at any time t be q. Then the pot. difference $V_C = V_a - V_b$ across the capacitor is given by,



$$V_c = \frac{q}{c}.$$

Now $i = \frac{dq}{dt}$, $dq = i\,dt.$

$$\int dq = \int i\,dt.$$

∴ The above equation becomes,

$$V_c = \frac{\int i\,dt}{C}.$$

$$\therefore \quad V_c = \frac{\int i_m \sin(wt - \varphi)\,dt}{C} \qquad \text{as } i = i_m \sin(wt - \varphi) \quad (1)$$

$$= -\frac{i_m \cos(wt - \varphi)}{Cw}.$$

$$V_c = -\frac{i_m}{Cw} \cos(wt - \varphi)$$

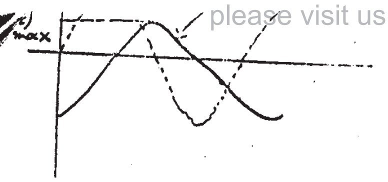
$$\because \sin(\theta - \pi/2) = -\cos\theta.$$

$$\therefore \boxed{V_c = \frac{i_m}{wc} \sin(wt - \varphi - \pi/2)} \qquad (2)$$

From (1) and (2) we find that i and V_c are not in phase. It is seen from (1) and (2) that voltage V_c lags behind current i by $\pi/2$ i.e i leads V_c by $\pi/2$. Fig (i) shows the phase relationship. It should be noted that i reaches its maximum value one quarter cycle or 90° before V_c.

So current leads the potential difference by 90° in a capacitor.

Fig (i)



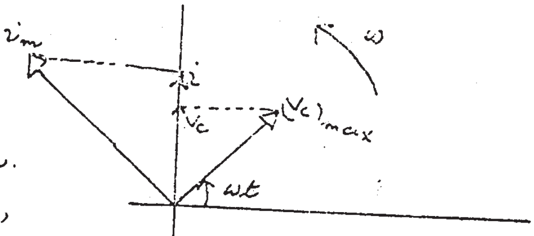


Fig (ii)

Fig. (ii) shows the phasor diagram. As the phasor rotates anticlockwise, it is clear that phasor i leads the phasor V_c by one quarter cycle.

Capacitive Reactance (X_C)

The term $\frac{1}{\omega C} = X_C$ is called capacitive reactance. It is the opposition offered by a capacitor to the flow of current. So equation (2) can be written as,

$$V_c = i_m X_C \sin(\omega t - \varphi - \pi/2)$$

The max. value of V_c is given by

$$(V_c)_{max} = i_m X_C \qquad \because (\sin\theta)_{max} = 1.$$

Power Dissipated in a Capacitor:

The instantaneous power dissipated in a capacitor is given by,

$$P = V_c\, i.$$

$$\therefore P = V_c\, i_m \sin(\omega t - \varphi) \qquad \text{As } i = i_m \sin(\omega t - \varphi)$$

From eq. (2) $\quad V_c = \frac{i_m}{\omega C} \sin(\omega t - \varphi - \pi/2)$

$$\therefore \boxed{P = \frac{i_m^2}{\omega C} \sin(\omega t - \varphi) \sin(\omega t - \varphi - \pi/2)}$$

Average Power Dissipated

Average power over a complete cycle is given by

$$\langle P \rangle = \frac{i_m^2}{\omega C} \langle \sin(\omega t - \varphi) \rangle \langle \sin(\omega t - \varphi - \pi/2) \rangle.$$

$$= \frac{i_m^2}{\omega C} (0).$$

$$\boxed{\langle P \rangle = 0.}$$

Energy stored in a Capacitor:

$$\text{As } P = \frac{dw}{dt}.$$

$$dw = P\,dt.$$

$$\text{But } P = V_c \times i = \frac{q}{C} \times \frac{dq}{dt}$$

$$\therefore dw = \frac{q}{C} \frac{dq}{dt} dt.$$

$$\boxed{dw = \frac{q}{C} dq.}$$

Integration gives

$$\int dw = \frac{1}{C}\int_0^q q\,dq.$$

$$w = \frac{1}{C}\left|\frac{q^2}{2}\right|_0^q$$

$$w = \frac{q^2}{2C}.$$

$\therefore$ Energy stored in the capacitor is given by

$$\boxed{E = \frac{1}{2}\frac{q^2}{C}.}$$

Sample Problem - 1

Sample Problem 1 In Fig. 4a let $L = 230$ mH, $\nu = 60$ Hz, and $(V_L)_{max} = 36$ V. (a) Find the inductive reactance X_L. (b) Find the current amplitude in the circuit.

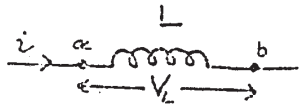
SOLUTION :

$L = 230\,mH = 230\times10^{-3}\,H$, $\nu = 60\,Hz$, $(V_L)_{max} = 36\,V.$

(a) $X_L = ?$, (b) $i_m = ?$

(a) As $X_L = \omega L$

$X_L = 2\pi\nu L$ $\qquad \because \omega = 2\pi\nu$



(Fig 4a)

$= 2\times 3.14\times 60\times 230\times10^{-3}$

$= 8664\times10^{-3} = 86.66$.

$\boxed{X_L = 87\,\Omega}$ Ans. $\qquad \because i = \frac{V}{R}.$

(b) As $i_m = \frac{(V_L)_{max}}{X_L} = \frac{36}{87}$

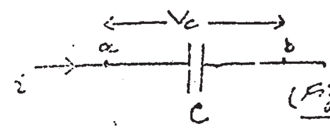
$\boxed{i_m = 0.41\,A.}$ Ans.

Sample Problem - 2

Sample Problem 2 In Fig. 5a let $C = 15\,\mu F$, $\nu = 60$ Hz, and $(V_C)_{max} = 36$ V. (a) Find the capacitive reactance X_C. (b) Find the current amplitude in this circuit.

SOLUTION: $C = 15\,\mu F = 15\times10^{-6}\,F$, $\nu = 60\,HZ$, $(V_C)_{max} = 36\,Volt$

(a) $X_c = ?$, (b) $i_m = ?$.



(Fig 5a)

(a) As $X_c = \frac{1}{\omega C}$

$\therefore X_c = \frac{1}{2\pi\nu C}$

$= \frac{1}{2\times3.14\times60\times15\times10^{-6}} = 1.769\times10^{-4}\times10^{6}$

$= 1.77\times10^{2}$

$\boxed{X_c = 177\,\Omega}$ Ans.

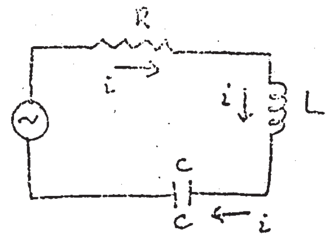
(b) As $i_m = (V_c)_{max}/X_c$.

$i_m = \frac{36}{177} = \boxed{0.20\ \text{Ampere.}}$ Ans.

The Single Loop R.L.C Circuit:

Consider an RLC series circuit as shown in fig. The sinusoidal emf is given by

$$E = E_m \sin\omega t.$$



The current in the circuit is given by,

$$i = i_m \sin(\omega t - \varphi).$$

We want to find i_m and φ.

∴ To solve the circuit we use loop theorem according to which

$$E - V_R - V_L - V_C = 0.$$

$$\text{or} \quad E = V_R + V_L + V_C \quad (1)$$

Equation (1) can be solved for i_m and φ by different ways. e.g
(i) Trignometric Analysis (ii) Graphical Analysis.
Now we discuss both the methods one by one.

(A) Trignometric Analysis:

As we know that

$$V_R = iR = i_m R \sin(\omega t - \varphi)$$

$$V_L = i_m X_L \sin(\omega t - \varphi + \pi/2) \text{ and } V_C = i_m X_C \sin(\omega t - \varphi - \pi/2)$$

Putting these values in (1) we get

$$E_m \sin\omega t = i_m R \sin(\omega t - \varphi) + i_m X_L \sin(\omega t - \varphi + \pi/2) + i_m X_C \sin(\omega t - \varphi - \pi/2)$$

$$\because \sin(\theta + \pi/2) = \cos\theta \text{ and } \sin(\theta - \pi/2) = -\cos\theta.$$

$$\therefore E_m \sin\omega t = i_m R \sin(\omega t - \varphi) + i_m X_L \cos(\omega t - \varphi) - i_m X_C \cos(\omega t - \varphi).$$

$$E_m \sin\omega t = i_m [R \sin(\omega t - \varphi) + (X_L - X_C)\cos(\omega t - \varphi)] \quad (2)$$

Multiplying and dividing R.H.S by $\sqrt{R^2 + (X_L - X_C)^2}$ we get,

$$E_m \sin\omega t = i_m \sqrt{R^2 + (X_L - X_C)^2} \left[\frac{R \sin(\omega t - \varphi)}{\sqrt{R^2 + (X_L - X_C)^2}} + \frac{(X_L - X_C)\cos(\omega t - \varphi)}{\sqrt{R^2 + (X_L - X_C)^2}} \right]$$

Putting $\dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \cos\varphi. \quad (a)$

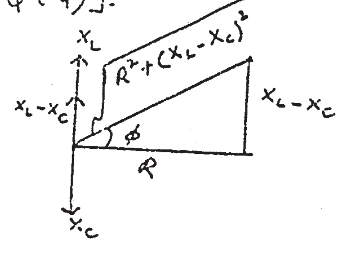
and $\dfrac{(X_L - X_C)}{\sqrt{R^2 + (X_L - X_C)^2}} = \sin\varphi$ - (b) we get.

$$E_m \sin\omega t = i_m \sqrt{R^2 + (X_L - X_C)^2} [\sin(\omega t - \varphi)\cos\varphi + \cos(\omega t - \varphi)\sin\varphi]$$

$$E_m \sin\omega t = i_m \sqrt{R^2 + (X_L - X_C)^2} [\sin(\omega t - \varphi + \varphi)].$$

$$E_m \sin\omega t = i_m \sqrt{R^2 + (X_L - X_C)^2} \sin\omega t$$

$$\boxed{i_m = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}}} \quad (3).$$



Now the quantity $\sqrt{R^2+(X_L-X_C)^2}$ is called the impedence Z of the series R.L.C circuit. It represents the total resistance of the circuit
∴ The above expression becomes

$$i_m = \frac{E_m}{Z}$$

The value of current i_m is maximum when the impedence is minimum. This happens when $X_L = X_C$

$$\text{i.e} \quad \omega L = \frac{1}{\omega C}.$$

$$\text{or } \omega^2 = \frac{1}{LC}.$$

$$\omega = \frac{1}{\sqrt{LC}}$$

As $\omega = 2\pi f$ $\quad \therefore f = \frac{\omega}{2\pi}$ $\quad$ So $f = \frac{1}{2\pi\sqrt{LC}}$ which is resonance condition.

Equation(3) gives value of i_m for any frequency. Resonance condition is a special case of eq.(3). Now for φ, we divide (b) by (a).

$$\therefore \frac{\sin\varphi}{\cos\varphi} = \frac{X_L - X_C}{\sqrt{R^2+(X_L-X_C)^2}} \times \frac{\sqrt{R^2+(X_L-X_C)^2}}{R}$$

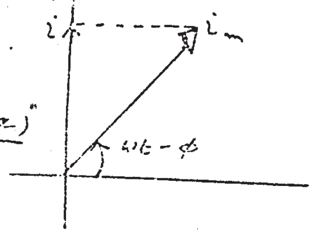
$$\tan\varphi = \left(\frac{X_L - X_C}{R}\right).$$

$$\boxed{\tan\varphi = \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)} \qquad (4)$$

Equation (4) shows that φ is independent of E_m. So by changing E_m, φ does not change. Equations (3) and (4) give the values of i_m and φ. So the analysis of RLC series circuits is completed.

(B) Graphical Analysis (Phasor Diagram)

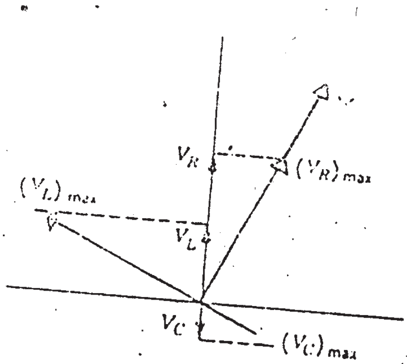
When A.C emf is applied to a RLC series circuit
(i) The current and voltage are in phase in resistance.
(ii) Voltage leads current by 90° in inductor and
(iii) Voltage lags behind current by 90° in capacitor.
Now we solve the RLC series circuit by phasor method.
Fig.(a) shows a phasor diagram representing current. The length of phasor represents i_m. Its projection on vertical axis gives current $i = i_m \sin(\omega t - \varphi)$ which is the time varying current.



(Fig a)

Fig.(b) shows phasor diagram representing individual potential differences across R, L and C.

From fig (b) we can note the max. values of V_R and V_L and V_C and their time varying values on the vertical axis.



In fig.(c) the vector sum of $(V_R)_{max}$, $(V_L)_{max}$ and $(V_C)_{max}$ gives a phasor of amplitude E_m.
In fig(c) we first form the vector sum of $(V_L)_{max}$ and $(V_C)_{max}$ which is a phasor (Fig b) $(V_L)_{max} - (V_C)_{max}$.
Next we form the vector sum of this phasor $(V_R)_{max}$.
Because $(V_R)_{max}$ is $\perp$ to $[(V_L)_{max} - (V_C)_{max}]$

$\therefore$ Amplitude of resultant phasor is E_m and is given by

$$(E_m)^2 = [(V_R)_{max}]^2 + [(V_L)_{max} - (V_C)_{max}]^2$$

$$\therefore \quad E_m = \sqrt{[(V_R)_{max}]^2 + [(V_L)_{max} - (V_C)_{max}]^2}$$

$\because \quad (V_R)_{max} = i_m R; \quad (V_L)_{max} = i_m X_L$ and $(V_C)_{max} = i_m X_C$.

$\therefore$ The above expression becomes.

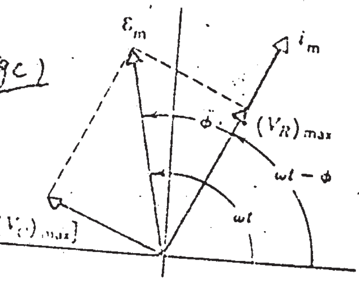
$$E_m = \sqrt{(i_m R)^2 + (i_m X_L - i_m X_C)^2}$$

$$= \sqrt{i_m^2 \quad R^2 + (X_L - X_C)^2}$$

$$E_m = i_m \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad \boxed{i_m = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}}} \qquad (1).$$

From figure (C)



(Fig c)

$$\tan\phi = \frac{(V_c)_{max} - (V_c)_{max}}{(V_R)_{max}}$$

$$= \frac{i_m X_L - i_m X_C}{i_m R}$$

$$\tan\phi = \frac{i_m(X_L - X_C)}{i_m R} = \frac{X_L - X_C}{R}.$$

$$\tan\varphi = \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \qquad (2).$$

It should be noted that the equations (1) and (2) are identical to the equations (3) and (4) obtained by trignometric method.
At resonance $X_L = X_C$ and $\varphi = 0$.
In this case $(V_L)_{max}$ and $(V_C)_{max}$ become equal and opposite and so i_m and E_m become in phase.

5. Power Dissipation in R.L.C Series Circuit:

The instantaneous power dissipated is

$$P = Ei.$$

$\because E = E_m \sin(\omega t)$ and $i = i_m \sin(\omega t - \varphi)$.

$$\therefore P = (E_m \sin\omega t)(i_m \sin(\omega t - \varphi))$$
$$= E_m i_m \sin\omega t \, \sin(\omega t - \phi).$$
$$= E_m i_m \sin\omega t \, (\sin\omega t \cos\varphi - \cos\omega t \sin\varphi)$$
$$= E_m i_m [\sin^2\omega t \cos\varphi - \sin\omega t \cos\omega t \sin\phi]$$

The average power over a complete cycle is

$$\langle P \rangle = E_m i_m [\langle \sin^2\omega t \rangle \cos\varphi - \langle \sin\omega t \cos\omega t \rangle \sin\varphi].$$

$$\because \langle \sin^2\omega t \rangle = \frac{1}{2} \text{ and } \langle \sin\omega t \cos\omega t \rangle = 0.$$

$$\therefore \langle P \rangle = E_m i_m [\tfrac{1}{2} \cos\varphi - 0]$$

$$\boxed{\langle P \rangle = \frac{1}{2} E_m i_m \cos\varphi}$$

$$\because E_{rms} = \frac{E_m}{\sqrt{2}} \text{ and } i_{rms} = \frac{i_m}{\sqrt{2}}.$$

$$\therefore \langle P \rangle = \frac{E_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} \cos\varphi.$$

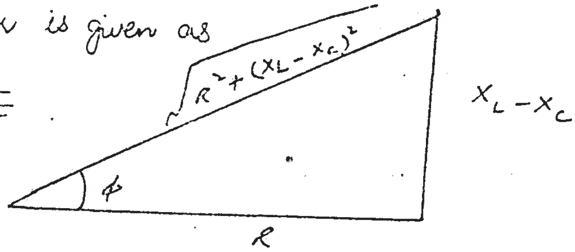
$$\boxed{\langle P \rangle = E_{rms} \, i_{rms} \cos\varphi} \quad (1).$$

Power Factor: The quantity $\cos\varphi$ in the above expression (1) is called power factor.

In R.L.C. series circuit powerfactor is given as

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\cos\phi = \frac{R}{Z}$$



According to eq. (1) average power dissipated is maximum when $\cos\varphi$ is maximum. But $(\cos\varphi)_{max} = 1$.

$$\cos\varphi = 1.$$
$$\varphi = \cos^{-1}(1).$$
$$\varphi = 0^\circ.$$

This happens when the circuit is purely resistive and contains no inductor or capacitor or at resonance $X_L = X_C$ and $R = Z$.

In this case

$$\boxed{\langle P \rangle = E_{rms} \, i_{rms}}$$

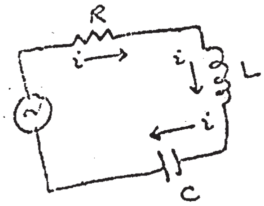
Sample Problem-3

Sample Problem 3 In Fig. 2 let $R = 160\ \Omega$, $C = 15\ \mu F$, $L = 230$ mH, $\nu = 60$ Hz, and $\mathcal{E}_m = 36$ V. Find (*a*) the inductive reactance X_L, (*b*) the capacitive reactance X_C, (*c*) the impedance Z for the circuit, (*d*) the current amplitude i_m, and (*e*) the phase constant ϕ.

SOLUTION: $R = 160\Omega$, $C = 15\mu F = 15\times10^{-6}F$.

$L = 230mH = 230\times10^{-3}H$. $\nu = 60Hz$, $\mathcal{E}_m = 36V$.

(i) $X_L = ?$, (ii) $X_c = ?$, (iii) $Z = ?$, (iv) $i_m = ?$, (V) $\varphi = ?$



(Fig 2)

(i) As $X_L = \omega L = 2\pi\nu L = 2\times3.14\times60\times230\times10^{-3}$

$\boxed{X_L = 87\Omega}$ Ans.

(ii) $X_c = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C} = \dfrac{1}{2\times3.14\times60\times15\times10^{-6}}$

$\boxed{X_c = 177\Omega \text{ Ans}}$

(iii) As $Z = \sqrt{R^2 + (X_L - X_c)^2}$

$= \sqrt{(160)^2 + (87-177)^2} = \boxed{184\Omega}$ Ans.

(iv) As $i_m = \dfrac{\mathcal{E}_m}{Z} = \dfrac{36}{184}$

$\boxed{i_m = 0.196 A}$ Ans.

(V) As $\tan\phi = \dfrac{X_L - X_c}{R} = \dfrac{87-177}{160} = 0.563$

$\tan\varphi = -0.563$

$\varphi = \tan^{-1}(-0.563)$

$\boxed{\varphi = -29.4^\circ}$ Ans.

Sample Problem-4

Sample Problem 4 (*a*) What is the resonance frequency in Hz of the circuit of Sample Problem 3? (*b*) What is the current amplitude at resonance?

SOLUTION: (i) Resonant frequency in sample Prob.(3) = ?

(ii) Current Amplitude at resonance = ?

From S.P(2)

$L = 230\times10^{-3}H$.

$C = 15\times10^{-6}F$.

$\mathcal{E}_m = 36V$, $R = 160\Omega$

(a) As $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{230\times10^{-3}\times15\times10^{-6}}}$

$\omega = \dfrac{1}{\sqrt{3450\times10^{-9}}} = \dfrac{1}{\sqrt{345\times10^{-8}}}$

$= \dfrac{1}{\sqrt{345}\times10^{-4}} = \dfrac{10^4}{18.57} = 0.0538\times10^4$

$\omega = 538 \times$ rad/sec

$\because \omega = 2\pi\nu$.

$\therefore \nu = \dfrac{\omega}{2\pi} = \dfrac{538}{2\times3.14}$

$\boxed{\nu = 0.86 \ Hz}$ Ans.

(b) At resonance $Z = R$.

$$\therefore \quad i_m = \frac{\mathcal{E}_m}{R} = \frac{36}{160}$$

$$\boxed{i_m = 0.23\ A} \quad \text{Ans}$$

Sample Problem 5

Sample Problem 5 Consider again the circuit of Fig. 2, using the same parameters that we used in Sample Problem 3, namely, $R = 160\ \Omega$, $C = 15\ \mu F$, $L = 230$ mH, $\nu = 60$ Hz, and $\mathcal{E}_m = 36$ V. Find (a) the rms emf, (b) the rms current, (c) the power factor, and (d) the average power dissipated in the resistor.

SOLUTION: $R = 160\ \Omega$, $C = 15\times10^{-6} F$, $L = 230\times10^{-3} H$, $\nu = 60$ Hz.

$\mathcal{E}_m = 36$ Volt

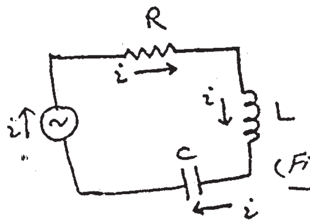
(i) $\mathcal{E}_{rms} = ?$, (ii) $i_{rms} = ?$, (iii) Power factor $= ?$

(iv) $\langle P \rangle$ in resistor $= ?$

From S. Prob 3.

$i_m = .196 A$.

$\varphi = -29.4°$.



(Fig 2)

(a) As $\mathcal{E}_{rms} = \frac{\mathcal{E}_m}{\sqrt{2}} = \frac{36}{1.414} = 25.5$

$$\therefore \boxed{\mathcal{E}_{rms} = 25.5\ \text{volt}} \quad \text{Ans.}$$

(b) As $i_{rms} = \frac{i_m}{\sqrt{2}} = \frac{0.196}{1.416} = 0.139 A$.

$$\boxed{i_m = 0.139\ \text{Ampere}} \quad \text{Ans.}$$

(c) As power factor $= \cos\varphi = \cos(-29.4°)$

$\therefore$ Power factor $= 0.871$ Ans.

(d) $\langle P \rangle = i_{rms}^2 R = (0.139)^2 \times 160$

$$\boxed{\langle P \rangle = 3.1\ \text{Watt}} \quad \text{Ans.}$$

6- The Transformer:

"It is the device used to increase or decrease A.C voltage in a circuit s.t the product of VI remains constant."

It works on the basis of mutual induction b/w the two coils.

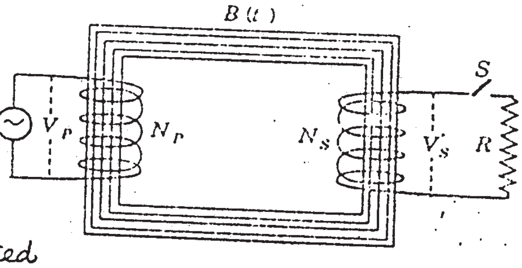
Construction: It consists of two coils of insulated cu wire wound on the same iron core. The core consists of soft iron strips piled upon each other seperated by thin layers of insulators. One coil of transformer connected to the (A.C) source is called Primary Coil. The number of turns in the primary coil is N_p. The emf of the (A.C) source is

$$\mathcal{E} = \mathcal{E}_m \sin\omega t.$$

The other coil across which voltage is induced is called secondary Coil. The number turns of secondary coil is N_s. The secondary coil is an open circuit as long as the switch 'S' is open or no load connected across it.

We also suppose that resistance of primary & secondary coil is neglegible

Case-I (When switch is open)



Working:

When the (A.C) voltage is given to the primary coil, the magnetic flux produced by the A.C current changes and an emf is induced in the primary coil. At the same time this magnetic flux due to A.C current also passes through the secondary coil. So a voltage is induced across the secondary coil also.

From Faraday's law of electromagnetic induction, the emf induced per turn ($E_T = \frac{d\phi_B}{dt}$) is the same both for primary and secondary coil because the primary and secondary fluxes are equal.

$$\text{i.e} \quad \left(\frac{d\phi_B}{dt}\right)_{Prim.} = \left(\frac{d\phi_B}{dt}\right)_{secondary.}$$

$$\text{or} \quad (E_{TURNS})_{Prim.} = (E_{Turns})_{secondary}$$

Now potential difference developed across the primary and secondary are given by,

$$V_p = N_p (E_{TURNS})_{Prim.} \qquad (1)$$

$$\text{and } V_s = N_s (E_{Turns})_{secon.} \qquad (2).$$

Dividing (2) by (1)

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}. \qquad (3)$$

$$V_s = V_p\left(\frac{N_s}{N_p}\right) \qquad (3).$$

Where V_p and V_s are r.m.s values of the voltages of primary and secondary coil respectively.

If $N_s > N_p$ then $V_s > V_p$. Then the transformer is called

Step up Transformer

If $N_s < N_p$ then $V_s < V_p$, then the transformer is called

Step down Transformer

The above results hold only when the switch S is open and no power is transmitted through the transformer.

Case-II (When switch is closed)

Now we connect a load 'R' in the secondary coil. Then we shall get an A.C power whose average value is $i_s^2 R = \frac{V_s^2}{R}$.

The A.C power flows from the primary coil to the secondary coil. As the transformer cannot create or destroy the electrical energy. So, for an ideal transformer

Input Power = Output Power

$$V_p i_p = V_s i_s$$

$$\frac{V_s}{V_p} = \frac{i_p}{i_s} \quad \text{———————} \quad (B)$$

Comparing (A) & (B) we get

$$\frac{i_p}{i_s} = \frac{N_s}{N_p} \quad \text{———————} \quad (C)$$

It shows that the ratio of current in the primary and secondary coil is equal to the inverse ratio of number of turns of the coils.

From equ. (C) $\quad i_s = i_p\left(\frac{N_p}{N_s}\right) \quad \text{———————} \quad (4)$

Now $\quad i_s = \frac{V_s}{R} \quad \text{———————} \quad (5)$

From equ. (4) & (5) $\quad i_p\left(\frac{N_p}{N_s}\right) = \frac{V_s}{R}$

$$i_p = \frac{V_s}{R}\left(\frac{N_s}{N_p}\right)$$

$$i_p = \frac{1}{R} V_s\left(\frac{N_s}{N_p}\right)$$

$$i_p = \frac{1}{R} V_p\left(\frac{N_s}{N_p}\right)\left(\frac{N_s}{N_p}\right) \qquad \because \text{From (3)} \; V_s = V_p\left(\frac{N_s}{N_p}\right)$$

$$i_p = \frac{V_p}{R}\left(\frac{N_s}{N_p}\right)^2$$

$$i_p = \frac{V_p}{R\left(N_p/N_s\right)^2}$$

It shows that equivalent resistance of load is not 'R' but

$$R_{eq} = R\left(N_p/N_s\right)^2$$

Applications

The uses of transformer cheapens the cost of the transmission of A.C. Power over long distances. We use a step up transformer at the power-house. A small current at high voltage is then carried to distant places by using thin copper wires which are cheaper than thick copper wires. A step down transformer is then used at the place where the electric power is to be used. This lowers the A.C voltage to a proper value (220 volts).

Losses in the Transformer:

The output power is equal to input power only in an ideal transformer having no loss.

In a practical transformer there are always some losses and its efficiency is less than 1. The losses in the transformer are listed below;

(i) Primary "Cu" Loss:

It is the loss due to heat produced in the resistance of the primary coil.

(ii) Secondary "Cu" Loss:

It is the loss due to heat produced in the resistance of the secondary coil.

(iii) Core Loss:

It is the loss of energy due to alternating flux in the core of the transformer.

Sample Problem - 6

Sample Problem 6 A transformer on a utility pole operates at $V_p = 8.5$ kV on the primary side and supplies electric energy to a number of nearby houses at $V_s = 120$ V, both quantities being rms values. The rate of average energy consumption in the houses served by the transformer at a given time is 78 kW. Assume an ideal transformer, a resistive load, and a power factor of unity. (*a*) What is the turns ratio N_p/N_s of this step-down transformer? (*b*) What are the rms currents in the primary and secondary windings of the transformer? (*c*) What is the equivalent resistive load in the secondary circuit? (*d*) What is the equivalent resistive load in the primary circuit?

Solution:

$$V_p = 8.5 \text{ KV} = 8.5 \times 10^3 \text{ V}$$

$$V_s = 120 \text{ V}$$

$$\bar{P} = 78 \text{ KW} = 78 \times 10^3 \text{ Watt}$$

(a) $\dfrac{N_p}{N_s} = ?$

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$= \frac{8.5 \times 10^3}{120}$$

$$\boxed{N_p/N_s = 70.8}$$ Ans.

(b) $i_p = ?$ $i_s = ?$

$$i_p = \frac{\bar{P}}{V_p}$$

$$= \frac{78 \times 10^3}{85 \times 10^3}$$

$$\boxed{i_p = 9.18 \text{ A}}$$ Ans.

$$i_s = \frac{\bar{P}}{V_s}$$

$$= \frac{78 \times 10^3}{120}$$

$$\boxed{i_s = 650 \text{ A}}$$ Ans.

(c) In the sec. circuit $R_s = ?$

$$R_s = \frac{V_s}{i_s}$$

$$= \frac{120}{650}$$

$$\boxed{R_s = 0.185 \ \Omega}$$ Ans.

(d) $R_p = ?$

$$R_p = \frac{V_p}{i_p}$$

$$= 8.5 \times 10^3 / 9.18$$

$$\boxed{R_p = 92.6 \ \Omega}$$ Ans