Nasir Pervaiz Butt
M. Sc. (Physics) M. Phil
Assistant Professor
Govt. College Sargodha.
House No. 338 St. No. 7
Cheema Colony,
Sargodha.

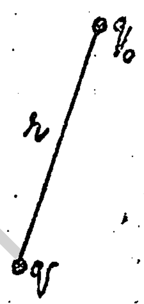
CH 28

ELECTRIC FIELD

"The region or area around a charge (or charges) in which they can exert forces on other charged bodies is called electric field."

1- Electric Field due to a point charge

Consider a point charge q_0 placed in the field of another point charge 'q' at a distance 'r' apart. The charge 'q' is source charge & charge 'q_0' is a test or field charge.



Now the force of repulsion b/w q & q_0 is given by coulombs law as

$$F = K\frac{qq_0}{r^2}$$

Now the electric field intesity at the test point is defined as "The force on unit +ve charge placed at field point"

Electric intensity at field point is given by

$$E = \frac{F}{q_0} \quad \text{———— (1)}$$

One thing should be noted that test charge should be very small so that it can not disturb the field produced by source charge.

$$E = \lim_{q_0 \to 0} \frac{F}{q_0}$$

However if the source charge is fixed at a certain position then the magnitude and the sign of the charge does not effect the result. So in calculating the electric fields we assume that source charges are fixed.

So by putting the value of "F" in equ (1) we

$$E = \frac{1}{q_0} \times F \quad \text{———— equ. 1}$$

$$= \frac{1}{q_0} \times \frac{K q q_0}{r^2}$$

$$E = \frac{Kq}{r^2}$$

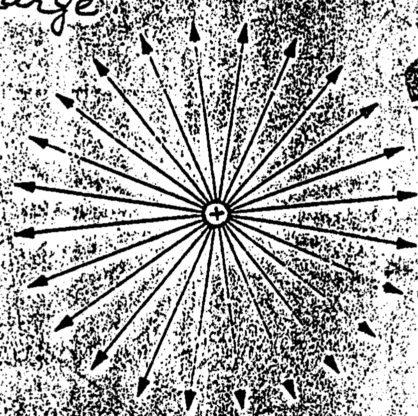
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \qquad \text{where } K = \frac{1}{4\pi\varepsilon_0}$$

This is the magnitude of electric intensity at the field point. In vector form it is given as;

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\, \hat{r}$$

Where $\hat{r}$ is a unit vector which gives the direction of electric field and it is directed from source charge to field charge.

It should be noted that electric field due to a point charge is radial and extends in all directions in space as shown in given fig.



Sample Problem-1

A proton is placed in a uniform electric field E. What must be the magnitude and direction of this field if the electrostatic force acting on the proton is just to balance its weight?

Solution

$q = 1.6 \times 10^{-19} C$

$m = 1.67 \times 10^{-27} kg$

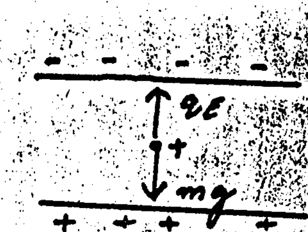
(given) – $F = mg$ ———— (i)

As $F = qE$ ———— (ii)

So from equ. (i) & (ii)

$qE = mg$

$E = mg/q$



$$= \frac{1.67\times10^{-27} \times 9.8}{1.6\times10^{-19}}$$

$$= \frac{1.67 \times 9.8}{1.6} \times 10^{-27+19}$$

$$= 10.2 \times 10^{-8}$$

$$E = 1.0 \times 10^{-7}\ N/C \quad \underline{\text{Ans.}}$$

2- Electric Field due to many (N) point charges.

Consider many point charges $q_1, q_2, q_3 \ldots\ldots, q_N$ then the electric intensity due to all these charges at the field point is calculated as follows;

1) Calculate electric intensity at given field point due to each charge separated assuming other charges as absent.

2) Calculate the total electric intensity at the field point by taking the vector sum of all the intensities calculated separately.

$\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots\ldots, \vec{E}_N$ be the intensities at given field point due to charges $q_1, q_2, \ldots\ldots, q_N$.

Then by the superposition principle, the total electric intensity at the given field point is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots\ldots + \vec{E}_N$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\hat{r}_2 + \ldots\ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_N}{r_N^2}\hat{r}_N$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_N}{r_N^2}\hat{r}_N\right)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i \quad \text{——— } (1)$$

This equ. (1) gives the vector sum taken over all the charges. This equation is an application of the principle of superposition.

4

Conditions for failure of Supeposition Principle

It should be noted that the principle of superposition may fail when the electric fields are extremely charge. However for the small fields this principle is valid.

Sample Prolem - 2

Sample Problem 2 In an ionized helium atom (a helium atom in which one of the two electrons has been removed), the electron and the nucleus are separated by a distance of 26.5 pm. What is the electric field due to the nucleus at the location of the electron?

Sol.

Total charge in the neucleus of helium ion = 2e

$$q = 2e$$

$$q = 2 \times 1.6 \times 10^{-19} C$$

$$r = 26.5 \text{ Pm} = 26.5 \times 10^{-12} m$$

$$E = ?$$

As we know $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$$E = 9\times10^9 \times \frac{2\times1.6\times10^{-19}}{26.5\times10^{-12} \times 26.5\times10^{-12}}$$

$$E = \frac{9\times2\times1.6}{26.5\times26.5} \times 10^{9-19+12+12}$$

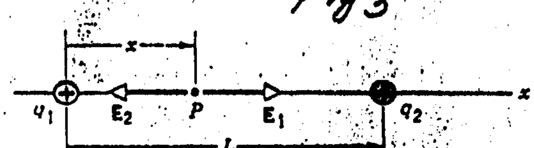
$$E = 0.041 \times 10^{14}$$

$$\boxed{E = 4.1\times10^{12} N/C}$$ Ans.

Sample Problem - 3

Figure 3 shows a charge q_1 of $+1.5\,\mu C$ and a charge q_2 of $+2.3\,\mu C$. The first charge is at the origin of an x axis, and the second is at a position $x = L$, where $L = 13$ cm. At what point P along the x axis is the electric field zero?

Fig 3



Sol.

$q_1 = +1.5\,\mu C = 1.5\times10^{-6} C$

$q_2 = +2.3\,\mu C = 2.3\times10^{-6} C$

$L = 13$ cm $= 0.13$ m

$x = ?$
For zero electric intensity at 'P'

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(L-x)^2}$$

$$\frac{q_1}{x^2} = \frac{q_2}{(L-x)^2}$$

$$q_1(L-x)^2 = q_2 x^2$$

$$1.5\times10^{-6}(0.13-x)^2 = 2.3\times10^{-6}x^2$$

$$1.5(0.13-x)^2 = 2.3x^2$$

$$(0.13-x)^2 = \frac{2.3}{1.5}x^2$$

$$(0.13-x)^2 = 1.533x^2$$

(By taking sq. root) $0.13 - x = 1.238x$

$$0.13 = 1.238x + x$$

$$= (1.238+1)x$$

$$0.13 = 2.238x$$

$$x = \frac{2.238}{0.13}$$

$$x = 0.058 \text{ m}$$

$$\boxed{x = 5.8 \text{ cm}}$$ Ans.

3. Electric Dipole:

"An electric dipole is defined as two equal and opposite charges separate by a small distance".

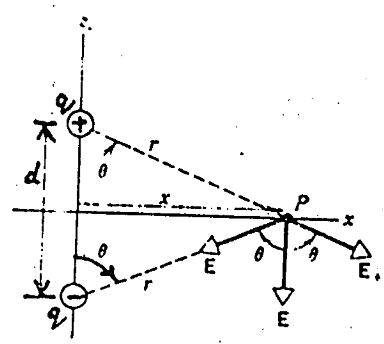
Electric Field due to Dipole:

Consider two point charges $+q$ and $-q$ of equal magnitudes lying 'd' distan apart as shown

We want to determine electric intensity $\vec{E}$ due to dipole at point 'P'.

Point 'p' is at a distance 'x' along the perpendicular bisector of the line joining the charges.

Let the intensities at 'P' due to $+q$ and $-q$ be $\vec{E}_+$ & $\vec{E}_-$. Now at point $\vec{E}_+$ and $\vec{E}_-$ have the same magnitude because point 'p' is equidistant from $+q$ and $-q$. The directions of $\vec{E}_+$ and $\vec{E}_-$ are shown in fig. This direction of $\vec{E}_+$ and $\vec{E}_-$ is along the force due to each charge on unit +ve charge placed at the point 'P'.



Now the total electric intensity is given by

$$\vec{E} = \vec{E}_+ + \vec{E}_- \qquad (\text{At point } P)$$

The magnitude of either intensity $\vec{E}_+ = \vec{E}_-$ is given by

$$\vec{E}_+ = \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

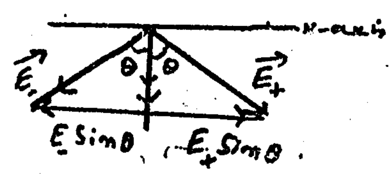
$$\vec{E}_+ = \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2+\left(\frac{d}{2}\right)^2} \quad \text{—— (1)}$$

Because $\vec{E}_+$ & $\vec{E}_-$ have same magnitudes and make same angle θ w.r.t. z-axis. Here x-component of $\vec{E}_+$ and $\vec{E}_-$ cancel away while z-components are added up. The x-component of total intensity is given by

$$E_+ \sin\theta - E_- \sin\theta = 0$$

and

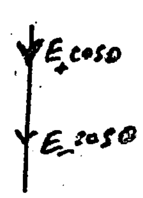
The z-component of the total intensity is given by



$$E = E_+\cos\theta + E_-\cos\theta$$

$$= E_+\cos\theta + E_+\cos\theta \qquad (\because E_+ = E_-)$$

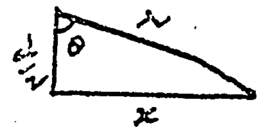
$$= 2E_+\cos\theta$$



$$E = 2E_+\cos\theta \quad \text{—— (2)}$$

L = 13 cm = 0.13 m

Now from the fig.



$$\cos\theta = \frac{d/2}{r}$$

$$\cos\theta = \frac{d/2}{\sqrt{x^2+(d/2)^2}}$$

Equ. ② becomes

$$E = 2E_+\left(\frac{d/2}{\sqrt{x^2+(d/2)^2}}\right)$$

$$E = E_+\left(\frac{d}{\sqrt{x^2+(d/2)^2}}\right)$$

By putting the value of E_+ from equ. ① we get

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2+\left(\frac{d}{2}\right)^2}\left(\frac{d}{\sqrt{x^2+(d/2)^2}}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left(x^2+\left(\frac{d}{2}\right)^2\right)^{3/2}} \longrightarrow ③$$

Now the quantity "$qd = P$" is called the Dipole moment. Its SI-unit is coulomb metre.

$$\therefore\ E = \frac{1}{4\pi\varepsilon_0}\frac{P}{\left[x^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{P}{x^3\left[1+\left(\frac{d}{2}\right)^2\frac{1}{x^2}\right]^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{P}{x^3}\frac{1}{\left[1+(d/2)^2\left(\frac{1}{x}\right)^2\right]^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{P}{x^3}\left[1+\left(\frac{d}{2x}\right)^2\right]^{-3/2}$$

By using bionomial expansion, we get

$$E = \frac{1}{4\pi\varepsilon_0}\frac{P}{x^3}\left[1+\left(\frac{-3}{2}\right)\left(\frac{d}{2x}\right)^2+\cdots\right]$$ if we neglect the 2nd Term then

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{P}{x^3}}$$

This is the expression for electric field due to dipole

Sample Problem - 4

In Fig. 5, how does the magnitude of the electric field vary with the distance from the center of the charged body?

Solution:

Suppose that N electric lines end on a spher of fig. We draw an imaginary concentric sphere of radius 'R'.

As number of lines of force passing normally per unit area gives the electric intensity. So the lines of force per unit area is ; $\frac{N}{4\pi r^2}$

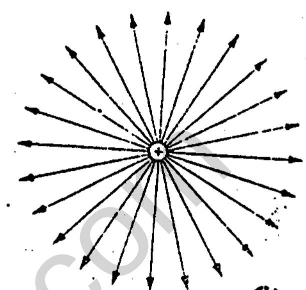


Fig 5

$$\therefore \quad E = \frac{N}{4\pi} \times \frac{1}{r^2}$$

$$\therefore E \propto \frac{1}{r^2}$$

Thus the electric field lines by +ve point charge is inversly proportional to the square of the distance from the centre of the sphere.

x————————x

4- Electric Field of Continuous Charge Distribution

"The collection of a large number of charges (elementary charges) is called continuous charge distribution".

The electric field due to continuous charge distribution can be calculated at any point 'p' by dividing the charge distribution infinitenimal (small) elements of charge 'dq'. Then each element of charge produces an electric field $d\vec{E}$ at point 'P'. It is then totaly found by using the principle of superposition i.e we add (integrate) all the fields due to all element as

$$\vec{E} = \int d\vec{E} \quad \text{———} \quad (1)$$

The integral shows summation. When charge distribution becomes continuous then sigma "Σ" is replaced by integral "$\int$". So equ. ① gives the vector sum. In cartesian co-ordinate system, we can write

$$E_x = \int dE_x \quad , \quad E_y = \int dE_y \quad , \quad E_z = \int dE_z$$

In general; the electric field at any point "p" due to charge distribution is calculated by taking an element of charge dq. We find the field $d\vec{E}$ at point "p" due to this element and integrate it over the entire distribution to get the total field $\vec{E}$.

We suppose that the charge contained in element is a point charge i.e its magnitude is so small. The magnitude of electric field due to this element is given as;

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} \quad \text{———} ②$$

Where "r" is the distance of the charge element dq from the field point "p".

In continuous charge distribution, we introduce the concept of charge density as

(1) **Linear Charge Density** (λ)

When charge is distributed along a line, then we define linear charge density (λ) as;

$$\lambda = dq/ds \quad (\text{where } \lambda \text{ is linear charge density})$$

or $\quad dq = \lambda\, ds$ ——— (i)

It is the charge per unit length of object. Here ds the element of length carrying charge dq. If the object is uniformally charged i.e charge is distributed uniformly over the object then λ is constant and is equal to total charge "q" on the object divided by its total length L.

$$\lambda = q/L$$

So expression (i) becomes

$$dq = \frac{q}{L}\, ds \quad \text{———} ⓐ \quad (\text{uniform linear charge})$$

(ii) Surface Charge Density (σ)

If a charge is distributed over a surface, then we define surface charge density as;

$$\sigma = \frac{dq}{dA}$$ (Where σ is surface charge density. i.e it is charge per unit are of the object.

(ii) ———— $dq = \sigma \, dA$

Here 'dA' is the element of area charge dq. If the object is uniformly charged i.e charge is distributed uniformly over the surface then 'σ' is constant and is equal to the total cha e 'q' divided by the total area A. of the surface

So $\sigma = \frac{q}{A}$

So expression (ii) becomes $dq = \frac{q}{A} dA$

(iii) Volume Charge Density (ρ)

When charge is distributed in a vol me, then we defi the volume charge density 'ρ' as;

$$\rho = \frac{dq}{dV}$$ (Where 'ρ' is called the volume cha e i.e it is the char per unit volume of e object).

or $dq = \rho \, dv$ ———— (iii)

Here dV is the element of the volume arrying charge dq. If the object is uniformly charged i.e charge is distributed uniformly ove the object then 'ρ' is constant is equal to the al charge 'q'. divided by total volume V.

So $\rho = \frac{q}{V}$

So expression (iii) becomes $dq = \frac{q}{V} dv$ ———— (C)

(Uniform Volu Charge).

Electric Field at a point due to An infinite-line of Charges

Consider a line of +ve charge along z-axis from −ve to +ve ∞. Fig shows the section of of this infinite line of charge. As the charge is distributed uniformly over it. So it has constant linear charge density 'λ'.

We want to calculate the electric field at a point 'P' at a perpendicular distance 'y' from the line of charge. Consider a small element dz of wire. If dq is the charge on this element. Then electric field $d\vec{E}$ due to this element at point 'P' is given by;

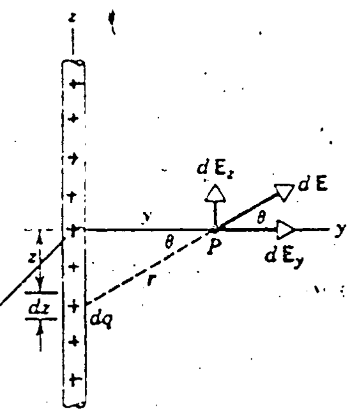
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$\because \lambda = dq/dz$

$dq = \lambda dz$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz}{r^2}$$

① ——— $$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz}{(y^2+z^2)}$$



The rectangular components of dE are

$$dE_y = dE\cos\theta \quad ——— ②$$

& $$dE_z = dE\sin\theta$$

If we consider identical charge elements symmetrically located on both sides of 'O', then z-components are canel away & y-components are added up. Hence the resultant field acts along y-axis is given by

$$E = E_y = \int_{z=-\infty}^{z=+\infty} dE_y$$

from② $dE_y = dE\cos\theta$

$$\therefore \quad E = \int_{z=-\infty}^{z=+\infty} dE\cos\theta$$

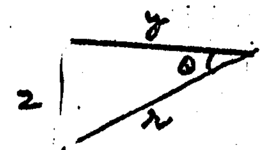
$$E = 2\int_{z=-\infty}^{z=+\infty} dE\cos\theta$$

By putting the values of dE from ①, we get

$$E = 2 \times \frac{1}{4\pi\varepsilon_0} \int_{z=0}^{z=\infty} \frac{\lambda\, dz}{(y^2+z^2)} \cos\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_0} \int_{z=0}^{z=\infty} \frac{dz \cos\theta}{(y^2+z^2)}$$

From the fig.



$$\frac{z}{y} = \tan\theta$$

$$z = y\tan\theta$$

$$dz = y\sec^2\theta\, d\theta$$

$$\therefore\ E = \frac{\lambda}{2\pi\varepsilon_0} \int_{z=0}^{z=\infty} \frac{y\sec^2\theta\, d\theta}{y^2+z^2} \cos\theta$$

$\because \sec\theta = \frac{1}{\cos\theta}$

$$E = \frac{\lambda}{2\pi\varepsilon_0} \int_{z=0}^{z=\infty} \frac{y\sec\theta\, d\theta}{y^2+z^2}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0} \int_{z=0}^{z=\infty} \frac{y\sec\theta\, d\theta}{y^2\left(1+\frac{z^2}{y^2}\right)}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y} \int_{z=0}^{z=\infty} \frac{\sec\theta\, d\theta}{(1+\tan^2\theta)}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y} \int_{z=0}^{z=\infty} \frac{\sec\theta\, d\theta}{\sec^2\theta}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y} \int_{z=0}^{z=\infty} \cos\theta\, d\theta$$

$\therefore \frac{1}{x} = \cos\theta$

Now $\theta = 0$ for $z = 0$

& $\theta = \pi/2$ for $z = \infty$

$\tan\theta.$
$\therefore y\tan\theta.$
$\tan$
$0.$
∞ . $-\infty$.
α .

$$E = \frac{\lambda}{2\pi\epsilon_0 r}\int_{\theta=0}^{\theta=\pi/2}\cos\theta\, d\theta$$

$$= \frac{\lambda}{2\pi\epsilon_0 r}\Big|\sin\theta\Big|_0^{\pi/2}$$

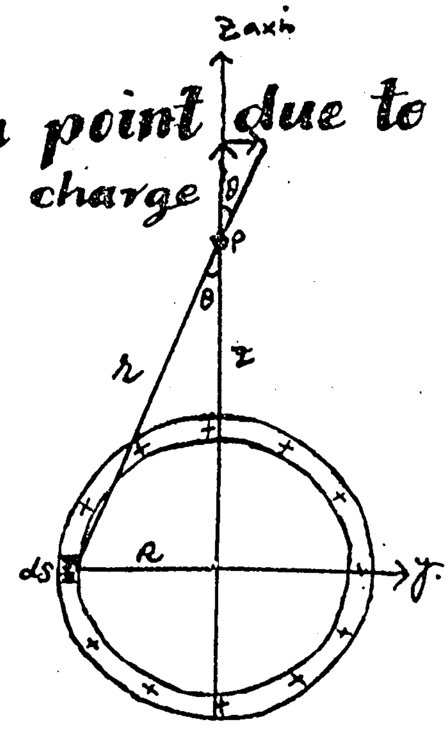
$$E = \frac{\lambda}{2\pi\epsilon_0 r}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}(1-0)$$

$$\boxed{E = \lambda/2\pi\epsilon_0 r}$$

This is the expression for electric field at a point 'P' due to infinite line of charge.

(b) Electric Field at a point due to A ring of charge



Fig

Consider a ring of +ve charge of radius 'R' having uniform linear charge density 'λ' around its circumference i.e charge is distributed uniformly over the of the ring.

Suppose that the ring is made up of some insulator, so that the charges are static.

We want to find the electric field at a point 'P' at a perpendicular distance 'z' from the plane of the ring.

Consider a small element of length ds of the ring.

If dq is the charge on this element, then electric intensity dE due to this element at point "P" is given by;

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$\lambda = \frac{dq}{ds}$

$dq = \lambda ds$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2+R^2)} \quad \text{———} (1)$$

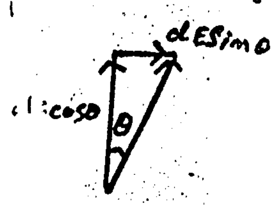
The rectangular components of dE a:z

$dE_z = dE \cos\theta$

$dE_y = dE \sin\theta$

If we consider identical charge ele ent located on the opposite end of diameter then ertical compo-nents cancel away and z-components re added up.

Hence the resultant field act alon; z-axis is given by;



$$E = E_z = \int dE_z$$

$$= \int dE \cos\theta$$

Putting the value of dE from (1), we g

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda ds \cos\theta}{(z^2+R^2)}$$

$$E = \frac{\lambda}{4\pi\epsilon_0} \int \frac{ds \cos\theta}{(z^2+R^2)}$$

$$E = \frac{\lambda}{4\pi\epsilon_0} \int \frac{ds}{r^2} \cdot \frac{z}{r}$$

$$= \frac{\lambda z}{4\pi\epsilon_0} \int \frac{ds}{r^3}$$

N. from fig.

$\frac{z}{r} = \cos\theta$

$z^2 + R^2 = r^2$

$$E = \frac{\lambda z}{4\pi\epsilon_0} \int \frac{ds}{(z^2+R^2)^{3/2}}$$

Now $z^2+R^2 = r^2$

$(z^2+R^2)^{1/2} = r$

$(z^2+R^2)^{3/2} = r^3$

$$E = \frac{z\lambda}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} \int ds$$

$$E = \frac{z\lambda}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} (2\pi R)$$

$\int ds$ = Total length of the ring

= circumference

$\int ds = 2\pi R.$

$$E = \frac{z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} (\lambda \times 2\pi R)$$

$$E = \frac{z q}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

λ = constt.

$$\lambda = \frac{\text{Total charge } q}{\text{Total length } 2\pi R}$$

$\lambda = q/2\pi R$

$\lambda \times 2\pi R = q$

When point 'P' is far away from the ring an $z >> R$. So R^2 can be neglected

$$E = \frac{z q}{4\pi\epsilon_0 (z^2)^{3/2}}$$

$$E = \frac{z q}{4\pi\epsilon_0 z^3}$$

$$E = \frac{q}{4\pi\epsilon_0 z^2}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}}$$

This is expression for electric field at a point "P" far away from the ring.

Note: If we put $z = R$ then

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

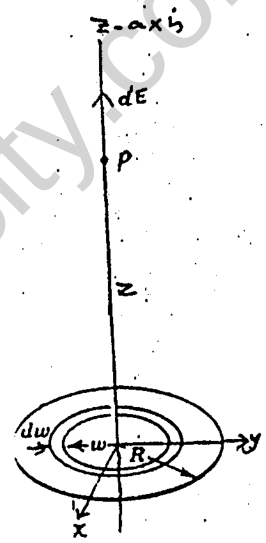
It mean acts as a point charge. Because at a large distance ring appears as a point charge.

Electric Field at a point due to "A Disk of Charge"

Consider a circular disk of +ve charge of radius 'R' having uniform surface charge density 'δ' on its upper surface. So that the disk is made up of some insulator so that the charges are static.

We want to find electric field at a point 'P' at a perpendicular distance 'z' from the of the disk-plane.

Consider a small element of the disk in the ring shape of radius 'w' and width 'dw'. If dq is the charge on this element of ring; then



$$\delta = \frac{dq}{dA}$$ (where dA is area of the ring-element)

$$dq = \delta\, dA$$

$$dq = \delta(2\pi w)\,dw \quad \text{———— ①} \qquad \because \quad dA = (2\pi w)\,dw$$

We know that electric field due to ring of charge is given by;

$$E = \frac{zq}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

Now we replace 'R' by 'w', 'q' by 'dq' ; 'E' by 'dE' in the above expression, we get

$$dE = \frac{z\,dq}{4\pi\epsilon_0 (z^2+w^2)^{3/2}}$$

By putting the value of 'dq' from equ. ① we get

$$dE = \frac{z\,\delta(2\pi w)\,dw}{4\pi\epsilon_0 (z^2+w^2)}$$

$$dE = \frac{\delta Z}{4\epsilon_0}(z^2+\omega^2)^{-3/2}(2\omega . d\omega)$$

Now total electric field at point 'P' due to whole the disk can be calculated by integrating the above expression from $\omega = 0$ to $\omega = R$

$$E = \int_{\omega=0}^{\omega=R} dE$$

$$= \frac{\delta Z}{4\epsilon_0}\int_0^R (z^2+\omega^2)^{-3/2}(2\omega . d\omega)$$

$$= \frac{\delta Z}{4\epsilon_0}\left|\frac{(z^2+\omega^2)^{-3/2+1}}{-3/2+1}\right|_0^R$$

$$= \frac{\delta Z}{4\epsilon_0}\left|\frac{(z^2+\omega^2)^{-1/2}}{-1/2}\right|_0^R$$

$$= -\frac{\delta Z}{2\epsilon_0}\left|\frac{1}{\sqrt{z^2+\omega^2}}\right|_0^R$$

$$= -\frac{\delta Z}{2\epsilon_0}\left(\frac{1}{\sqrt{z^2+\omega^2}} - \frac{1}{\sqrt{z^2+0}}\right)$$

$$= -\frac{\delta Z}{2\epsilon_0}\left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{z}\right)$$

$$= \frac{\delta Z}{2\epsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right)$$

$$= \frac{\delta \cancel{Z}}{2\epsilon_0}\left(\frac{z}{z} - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$= \frac{\delta}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

if $R >> Z$; $z = 0$

then $\frac{z}{\sqrt{z^2+R^2}} = \frac{0}{\sqrt{0+R^2}} = 0$

hence $$E = \frac{\delta}{2\epsilon_0}$$

It means when $z \approx 0$ the disk of charge behaves like infinite sheet of charge.

6. Point Charge in an Electric Field.

Consider a charged particle 'q' placed in an electric field of strength '$\vec{E}$'. Then the charged particle experiences force given by;

$$\vec{F} = q\vec{E}$$

To study the motion of charged particle in electric field, we have Newton's 2nd Law of motion

$$\Sigma \vec{F} = m\vec{a}$$

(where $\Sigma \vec{F}$ is the resultant force on the particle including electrostatic force and all other forces).

We suppose that electric field is uniform and electric force on the charged particle constant. This can be done by connecting the terminals of a battery to a pair of parallel metallic plates kept at a distance from each other. If the distance b/w the plates is small as compared to their dimensions then the field will be uniform in the middle while non uniform at the edges called ... field.

Sample Problem - 5

Sample Problem 5 A charged drop of oil of radius $R = 2.76\ \mu m$ and density $\rho = 920\ kg/m^3$ is maintained in equilibrium under the combined influence of its weight and a downward uniform electric field of magnitude $E = 1.65 \times 10^6$ N/C (Fig. 13). (*a*) Calculate the magnitude and sign of the charge on the drop. Express the result in terms of the elementary charge *e*. (*b*) The drop is exposed to a radioactive source that emits electrons. Two electrons strike the drop and are captured by it, changing its charge by two units. If the electric field remains at its constant value, calculate the resulting acceleration of the drop.

Solution:

$R = 2.76\ \mu m = 2.76 \times 10^{-6}\ m$

$\rho = 920\ Kg/m^3$

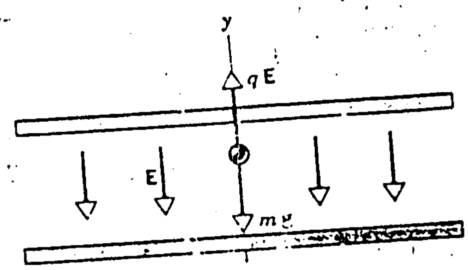
$E = 1.65 \times 10^6\ N/C$

$q = ?$

Sign of 'q' = ?

For the drop to be in equilibrium there must

equal and opposite force on the drop to balance its weight 'mg'.
As electric field is downward So electric force must be upward. So electric charge 'q' must be -ve.
By using 1st condition of equilibrium,



$$\Sigma \vec{F} = m\vec{g} + q\vec{E} = 0$$

Taking y-component

$$-mg + (-qE) = 0$$

$$qE = -mg$$

$$\therefore \quad q = \frac{-mg}{E}$$

$\frac{m}{V} = \rho$ (density)
$m = \rho V$
$m = \rho \frac{4}{3}\pi R^3$

$$q = -\frac{4\pi R^3 \rho g}{3E}$$

$$= -\frac{4 \times 3.14 \times (2.76 \times 10^{-3})^3 \times 920 \times 9.8}{3 \times 1.65 \times 10^6}$$

$$= -\frac{4 \times 3.14 \times 2.76 \times 2.76 \times 2.76 \times 920 \times 9.8 \times 10^{-18-6}}{\cancel{4.95}\ 3 \times 1.65}$$

$$= -480978 \times 10^{-24}$$

$$\boxed{q = -4.8 \times 10^{-19} \text{ Col}}$$

As $\quad q = -ne \quad$ (where 'n' is the no. of electronic charges on drop).

$$n = \frac{q}{-e}$$

$$= \frac{-4.8 \times 10^{-19}}{-1.6 \times 10^{-19}}$$

$$\boxed{n = 3}$$

(b) If we add two additional electrons to the drop then its charge will become

$$q' = (n+2)(-e)$$
$$= (3+2)(-e)$$
$$= -5e$$
$$= -5 \times (1.6 \times 10^{-19})$$
$$\boxed{q' = -8 \times 10^{-19} C}$$

According to the Newton's 2nd Law of motion

$$\sum \vec{F} = m\vec{a}$$

$$m\vec{g} + q'\vec{E} = m\vec{a}$$

Taking y-component

$$\frac{-m\vec{g} + (-q'\vec{E})}{m} = \vec{a}$$

OR $$a = -g - \frac{q'E}{m}$$

$$= -9.8 - \frac{(-8 \times 10^{-19}) \times 1.6 \times 10^{6}}{\frac{4}{3}\pi R^3 \rho} \qquad \because \frac{4}{3}\pi R^3 \rho = m$$

$$= -9.8 + \frac{8 \times 1.65 \times 3 \times 10^{-19+6}}{4 \times 3.14 \times (2.76 \times 10^{-6})^3 \times 920}$$

$$= -9.8 + \frac{39.6 \times 10^{5}}{242943}$$

$$= -9.8 + 1.63 \times 10^{-4} \times 10^{5}$$

$$= -9.8 + 1.63 \times 10$$

$$\boxed{a = +6.5 \; m/s^2}$$

So the drop accelerates in the +y-direction.

x————x

Sample Problem 6

Figure 14 shows the deflecting electrode system of an ink-jet printer. An ink drop whose mass m is 1.3×10^{-10} kg carries a charge q of -1.5×10^{-13} C and enters the deflecting plate system with a speed $v = 18$ m/s. The length L of these plates is 1.6 cm, and the electric field E between the plates is 1.4×10^{6} N/C. What is the vertical deflection of the drop at the far edge of the plates? Ignore the varying electric field at the edges of the plates.

Solution:

$m = 1.3\times10^{-10}\,kg$ $\qquad$ $q = -1.5\times10^{-13}$ coulombs.

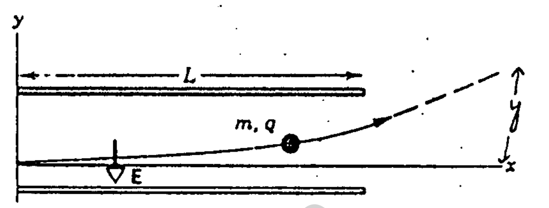
$V = 18\,m/s$ $\qquad$ $L = 1.6\,cm = 1.6\times10^{-2}\,m$

$E = 1.4\times10^{6}\,N/C$

Vertical deflection of the drop $y = ?$

As particle is thrown with constant velocity along x-axis

$\therefore\ L = Vt$ ——— (i)



$y = V_i t + \frac{1}{2}at^2$

$y = \frac{1}{2}at^2$ $\qquad \because$ Initial velocity is zero.

$y = \frac{1}{2}a\frac{L^2}{V^2}$ $\qquad \because$ from (1) $t = \frac{L}{V}$

$y = \frac{aL^2}{2V^2}$ ——— (ii)

By Newton's IInd Law

$-mg - qE = ma$

$\therefore\ -qE = ma$ $\qquad \because qE >> mg$

$a = \frac{-qE}{m}$

Putting the value of 'a' in equ. (ii)

$y = \frac{-qE}{m}\,\frac{L^2}{2V^2}$

$= \frac{-qEL^2}{2mV^2}$

$y = \frac{-(-1.5\times10^{-13})\times1.4\times10^{6}\times(1.6\times10^{-2})^2}{2\times(1.3\times10^{-10})\times(18)^2}$

$= \frac{1.5\times1.4\times1.6\times1.6}{2\times1.3\times18\times18}\times10^{-13+6-4+10}$

$$y = \frac{5.376}{842.4} \times 10^{-1}$$

$$= 6.38 \times 10^{-1} \times 10^{-3} = 0.638 \times 10^{-3}\,m.$$

$\boxed{y = 0.64\,mm}$ Ans.

Sample Problem: 7

Solution $P = 6.2 \times 10^{-30}\,cm$

(a) $d = ?$ In H_2O molecules, there are 10 electrons and 10 protons. $\therefore n = 10$

$\therefore P = qd$ $\quad \because q = ne.$

$= ned.$

$P = 10\,ed.$

$$d = \frac{P}{10e}$$

$$= \frac{6.2 \times 10^{-30}}{10 \times 1.6 \times 10^{-19}}$$

$$= \frac{6.2 \times 10^{-30+19}}{16}$$

$$= 0.387 \times 10^{-11}.$$

$$= 3.87 \times 10^{-12}$$

$$= 3.9 \times 10^{-12}\,m$$

$\boxed{d = 3.9\,Pm}$ Answer.

(b) $E = 1.5 \times 10^{4}\,N/c$, $\tau_{max} = ?$

As $\tau = PE\sin\theta$. As τ is max. when $\theta = 90°$

$\therefore \tau_{max} = PE\sin 90°.$

$$PE = 6.2 \times 10^{-30} \times 1.5 \times 10^{4} \times 1$$

$$= 6.2 \times 1.5 \times 10^{-30+4}$$

$\boxed{P.E = 9.3 \times 10^{-26}\,N\_m}$ Ans.

(c). $\theta_0 = 180°$, $\theta = 0°$, work $= ?$

The work done from rotating the dipole from $\theta_0 = 180°$ to $0°$ is given by

$$W = PE\,(-\cos\theta + \cos\theta_0)$$

$$= PE\,(-\cos 0° + \cos 180°)$$

$$= PE\,(-1 - 1)$$

$$W = -2PE.$$

Work being scalar is not $-$ve.

$$\therefore W = 2PE$$

$$= 2 \times 6.2 \times 10^{-30} \times 1.5 \times 10^{4}$$

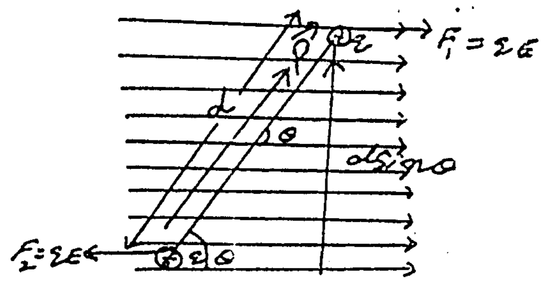
$\boxed{W = 1.9 \times 10^{-25}\,J}$ Ans.

Dipole in an Electric Field

Torque on a Dipole in a uniform Electric-Field

"Two equal and opposite charges +q and -q seperated by a small distance form an electricfield."

Consider an electric dipole of length 'd' placed in a uniform electric field of intensity $\vec{E}$. The axis of the dipole is making an angle θ with the electric field as shown.



The forces acting on +q and -q are equal in magnitude but opposite in direction. ∴ the net force on the dipole is zero. The magnitude of each force is

$$F_1 = F_2 = F = qE.$$

These two forces make a couple and so Torque acts on the dipole. The magnitude of this Torque is given as

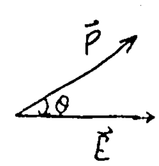
$$\tau = \text{Force} \times \text{couple arm}.$$
$$= F(d \sin\theta)$$
$$= qEd\sin\theta.$$
$$\tau = Eqd\sin\theta$$

But qd = P = Dipole moment. It is a vector quantity directed from -q to +q.

$$\therefore \tau = PE\sin\theta.$$

In vector form it is written as

$$\vec{\tau} = \vec{P} \times \vec{E}$$



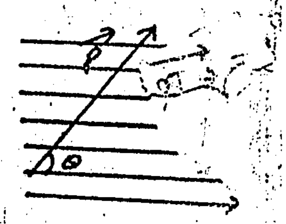
The direction of torque by right hand rule is into the plane of page.

Torque tends to rotate vector $\vec{P}$ along $\vec{E}$.

Energy of Dipole in a Uniform Electric Field:

Consider an electric dipole placed in a uniform electric field. We have to calculate the work done by the electric field in turning the dipole through an angle θ.

The work done by the electric field in turning the dipole from an initial angle θ_0 to the final angle θ is given by



$$W = \int dw.$$

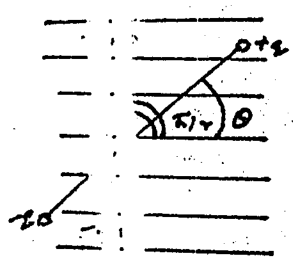
$$W = \int_{\theta_0}^{\theta} T\,d\theta$$

Where T is the torque exerted by the electric field

As we know that $T = PE\sin\theta$.

$$\therefore W = \int_{\theta_0}^{\theta} PE\sin\theta\,d\theta.$$

Now suppose the dipole is rotated from initial angle $\theta_0 = \pi/2$ to final angle θ.



$$W = \int_{\pi/2}^{\theta} PE\sin\theta\,d\theta.$$

$$= PE \int_{\pi/2}^{\theta} \sin\theta\,d\theta$$

$$= PE\,\left|-\cos\theta\right|_{\pi/2}^{\theta} = -PE\left|\cos\theta\right|_{\pi/2}^{\theta}$$

$$= -PE\,(\cos\theta - \cos\pi/2)$$

$$= -PE\cos\theta$$

$$= -\vec{P}.\vec{E}$$

This work done becomes the P.E of dipole.

$$\therefore \text{Potential energy} = \text{Work done}$$

$$\therefore U = -\vec{P}.\vec{E}$$

This is the expression for potential energy of dipole in a uniform electric field.