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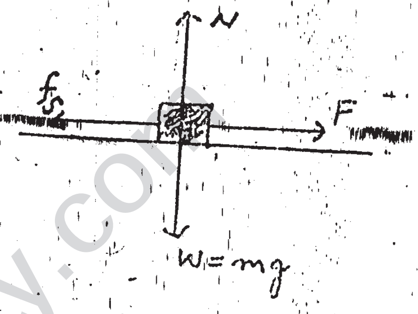
Chapter-2

PARTICLE DYNAMICS

1- Frictional Forces:

When body slides over the surface of another body, each body opposes the motion of the other. This opposition is called "friction force".
The direction of force of friction is opposite to the motion of the body (always).

Consider a block of mass "m" on a horizontal surface of a table as shown.
A force $\vec{F}$ is applied on the body, the block does not move if $\vec{F}$ is small. So $\vec{F}$ is balanced by the opposite friction force f exerted by the table surface on the block.



If the magnitude of force $\vec{F}$ is increased then for a certain value of $|F|$, the block will begin to move.

Static Friction:

The friction forces acting between the surfaces at rest w.r.t each other are called forces of "static friction". Static friction is denoted by "f_s".

The maximum value of static friction is called "Limiting Friction". Once, the motion is started, the force of friction is decreased.

Kinetic Friction:

The frictional forces acting b/w the surfaces in motion w.r.t. each other are called forces of "kinetic friction". Kinetic friction is denoted by "f_k".

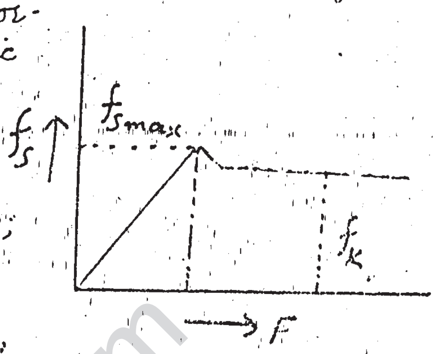
Laws of Static Friction:

The static friction obeys following two laws.

(i) It is independent of area of contact.
(ii) It is proportional to the normal reaction.

Co-efficient of Static Friction:

The ratio of magnitude of maximum force of static friction to the magnitude of normal reaction is called co-efficient of static friction. It is denoted by "μ_s".



i.e $\mu_s = \frac{f_s}{N}$

The magnitude of static friction f_s is;

$$f_s \leq \mu_s N$$

The sign of equality holds when "f_s" is maximum.

Laws of Kinetic Friction:

(i) It is independent of area of contact.
(ii) It is proportional to the normal reaction.

Co-efficient of Kinetic Friction:

The ratio of magnitude of force of kinetic friction to the magnitude of normal reaction is called co-efficient of kinetic friction. It is denoted by "μ_k".

i.e $\mu_k = \frac{f_k}{N}$

The magnitude of force of kinetic friction f_k is given as;

$$f_k = \mu_k N$$

It is to be noted that $\mu_k < \mu_s$. Both μ_k & μ_s depend on the nature of surfaces of two bodies. Both are large for rough surfaces and small for polished surfaces. μ_k and μ_s both are dimensionless.

Microscopic Basis of Friction:

When two bodies are placed incontact, the actual area of contact is less than the true area of the surfaces. The actual (microscopic) area of contact is proportional to the normal reaction. It is due to reason that the contact points are deformed due to stresses. Most of the contact points become cold welded due to surface adhesion. Surface adhesion occurs due to strong intermolecular forces of the two surfaces.

When one body is pulled across another, the force of friction ruptures thousands of these welds and new contacts are formed. In this process small fragments of one surface shear off and stick to the other surface.

For greater relative motion b/w two surfaces, the melting may occure at certain points. The noises that dry surfaces make during sliding is due to slip & stick.

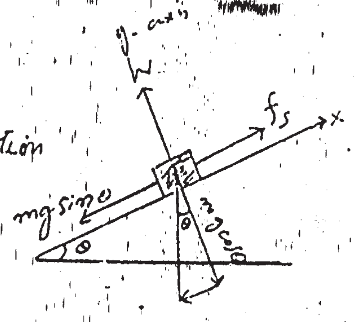
The coefficient of friction depends upon nature of materials, state of polish of two surfaces, temperature & extant of contamination. The fractional forces for rolling motion is less than that for sliding motion. Sliding friction can be reduced by the use of lubsicant ~~of~~ material. The affeciency of lubricants is due to the fact it holds the two surfaces some what apart from each other. So their inequalities can't interlock tightly.

Sample Problem 1

Sample Problem 1 A block is at rest on an inclined plane making an angle θ with the horizontal, as in Fig. 4a. As the angle of incline is raised, it is found that slipping just begins at an angle of inclination $\theta_s = 15°$. What is the coefficient of static friction between block and incline?

<u>Solution:</u>

$\theta_s = 15°$ $\quad \mu_s = ?$



As the block is at rest, By 2nd Law of Motion

$$\sum \vec{F} = 0$$

Along x-axis

$$\sum \vec{F}_x = f_s - mg\sin\theta = 0$$

or $\quad f_s = mg\sin\theta$ ——— (1)

Along y-axis;

$$\sum \vec{F}_y = 0$$

or $\quad N = mg\cos\theta$ ——— (2)

When $\theta = \theta_s$ Then equation (1) and equation (2) becomes;

$f_s = mg\sin\theta_s$ ——— (3)

$N = mg\cos\theta_s$ ——— (4)

Dividing equ. ③ by ④ we get

$$\frac{f_s}{N} = \frac{\cancel{mg}\,\sin\theta_s}{\cancel{mg}\,\cos\theta_s}$$

$$\frac{f_s}{N} = \tan\theta_s \qquad \text{But } \frac{f_s}{N} = \mu_s$$

$$\therefore \quad \mu_s = \tan\theta_s$$

$$\mu_s = \tan 15^\circ \qquad \theta_s = 15^\circ \text{ (given)}$$

$$\mu_s = 0.267$$

$$\boxed{\mu_s = 0.27} \text{ Ans.}$$

Sample Problem 2

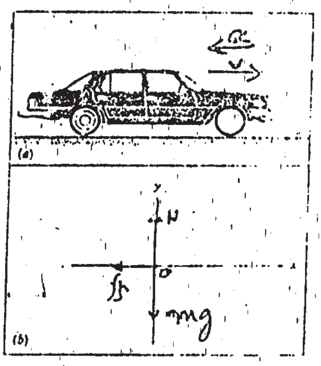
Sample Problem 2 Consider an automobile moving along a straight horizontal road with a speed v_0. If the coefficient of static friction between the tires and the road is μ_s, what is the shortest distance in which the automobile can be stopped?

Solution:

Initial speed $= V_0$

Co-efficient of static friction $= \mu_s$

Stopping distance $= x = ?$



By the equation of motion;

$$V_f^2 - V_i^2 = 2ax$$

$$0^2 - V_0^2 = 2ax$$

$$x = \frac{-V_0^2}{2a} \quad \text{——— ①}$$

For the value of 'a' we use 2nd law of motion along x-axis i.e

$$f_s = -ma$$

$$a = \frac{-f_s}{m}$$

$$a = -\frac{\mu_s \cancel{m} g}{\cancel{m}} \qquad \because f_s = \mu_s N = \mu_s mg.$$

$$\boxed{a = -\mu_s g}$$

∴ Putting 'a' in equ. ①, we get

$$x = \frac{-V_0^2}{2(-\mu_s g)}$$

$$\boxed{x = \frac{V_0^2}{2\mu_s g}}$$ Ans.

x can be found if we know V_0 and μ_s.
e.g, if $V_0 = 27\ m/s$ and $\mu_s = 0.6$ then

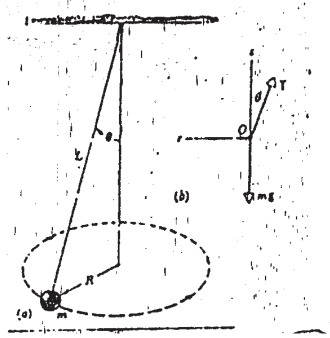
$$x = \frac{27 \times 27}{2 \times 0.6 \times 9.8}$$

$$= \frac{729}{11.76}$$

$$\boxed{x = 62\ m}$$ Ans.

3. The Dynamics of Uniform Motion

(i) The Conical Pendulum:



"A small body of mass 'm' suspended by a spring of length 'L' and revolving in a horizontal circle is called a conical pendulum."

As the body swings around, the string sweeps over a surface of an imaginary cone. We want to find the time period of pendulum.

If the string makes an angle 'θ' with the verticle, then radius 'R' of circular path is given by:

$$R = L \sin\theta \qquad \because \frac{R}{L} = \sin\theta$$

The forces acting on the body are

(i) weight of body 'mg' acting vertically downward.
(ii) Tension 'T' in the string acting upward along the string.
The rectangular components of 'T' are

$$T_r = -T\sin\theta \qquad \text{and} \qquad T_z = T\cos\theta$$

'T_r' is -ve because it is radially inward. As there is no motion of the body in the direction of 'T_z' and 'mg'.

$$\therefore \sum \vec{F}_z = 0$$

$$\text{i.e} \quad T_z = mg$$

$$\boxed{T\cos\theta = mg} \text{ ——— (1)} \qquad \because T_z = T\cos\theta$$

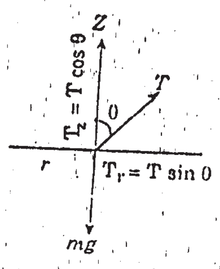
As the body is revolving in the circle so necessary centripetal force is provided by 'T_r'.

$$\therefore \sum F_r = T_r = ma_r$$

$$-T\sin\theta = -\frac{mV^2}{R} \qquad \because a_r = \frac{-V^2}{R}$$

$$\boxed{T\sin\theta = mV^2/R} \text{ ——— (2)}$$

Dividing equ. (2) by equ. (1), we get



$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{mV^2}{R} \div mg$$

$$\tan\theta = \frac{\cancel{m}V^2}{R} \times \frac{1}{\cancel{m}g}$$

$$\tan\theta = V^2/gR$$

$$V^2 = gR\tan\theta$$

$$\boxed{V = \sqrt{Rg\tan\theta}} \text{ ——— (3)}$$

Equ. (3) gives constant speed of the body. If 't' is the time period of the body then;

$$t = \frac{S}{V}$$

$$t = \frac{2\pi R}{V}$$

Putting 'v' from equ ③, we get

$$t = \frac{2\pi R}{\sqrt{Rg\,Tan\theta}}$$

$$t = \frac{2\pi\sqrt{R^2}}{\sqrt{Rg\,Tan\theta}}$$

$$t = \frac{2\pi\sqrt{R}}{\sqrt{g\,Tan\theta}}$$

$$t = 2\pi\sqrt{\frac{R}{g\,Tan\theta}}$$

$$t = 2\pi\sqrt{\frac{L\,Sin\theta}{g\,Tan\theta}} \qquad \because R = L\,Sin\theta$$

$$t = 2\pi\sqrt{\frac{L}{g}\cos\theta} \qquad \because \frac{Sin\theta}{Tan\theta} = \frac{Sin\theta}{Sin\theta/\cos\theta} = \cos\theta$$

The equation gives time period of conical pendulum. So we find that time period is independent of mass of the pendulum.

x———————x

Problem:

Calculate the time period of a conical pendulum for which $L = 1.2m$ $\theta = 25^\circ$

<u>Solution</u>:

As $t = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

Putting the values in above expression, we get

$$t = 2\times 3.14\sqrt{\frac{1.2\times\cos 25^\circ}{9.8}}$$

$$= 6.28\sqrt{\frac{1.2\times 0.906}{9.8}}$$

$$= 6.28\sqrt{0.1109}$$

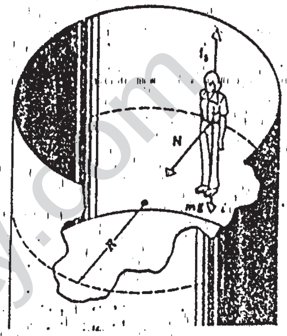
$$t = 6.28\times 0.33$$

$$\boxed{t = 2.09\,sec.}\ \text{Ans.}$$

(ii) The Rotor:

The rotor is a hollow cylindrical shell capable of rotating about the central vertical axis.
In many amusement parks, we can find this rotor.

A person enters the rotor, closes the door, and stands up against the wall. The rotor is then sets into rotation. At a particular predetermined speed, the floor be-low his feet is opened downward.
The person does not fall but rema-ins clung against the wall of rotor. The minimum speed of rotation necess-ary to prevent falling can be calcu-lated as follows.



The forces acting on the person are shown in figure.

(i) Weight of the person "mg" acting vertically downward

(ii) Force of static friction b/w the man and rotor wall acting upward whi-ch prevents the man from falling.

(iii) The normal reaction N of the wall of rotor on the man which provides necessary centripetal force. If z axis is in upward direction and the man does not fall. Then the net force along z axis is;

$$\sum F_z = 0$$

i.e $\quad f_s - mg = 0$

$$\boxed{f_s = mg} \quad \text{———} \ (1)$$

If "R" is the radius of rotor then

$$\sum F_r = ma_r$$

i.e $\quad -N = \dfrac{-mv^2}{R}$

$$\boxed{N = \frac{mv^2}{R}} \quad \text{———} \ (2)$$

If μ_s be be the coefficient of static friction b/w the man and the wall, then

$$f_s = \mu_s N$$

From equ: ① $\quad f_s = mg$

$$\therefore \mu_s N = mg$$

From equ. ② $\quad N = \frac{mv^2}{R}$

$$\therefore \mu_s\left(\frac{mv^2}{R}\right) = mg$$

$$\mu_s \frac{v^2}{R} = g$$

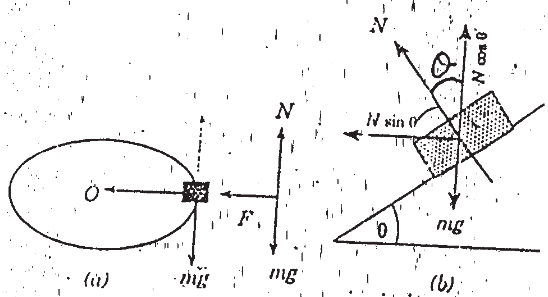
$$v^2 = Rg/\mu_s$$

$$\boxed{v = \sqrt{Rg/\mu_s}}$$

This gives the maximum speed of rotor necessary to prevent falling. This equ. gives a relation b/w co-efficient of static friction and tangential speed 'v' to prevent slipping.
We find that 'v' is independent of weight of the person.

(iii) The Banked Curve

A track is said to be banked if its outer edge is raised w.r.t inner edge.
Consider a car moving with constant speed 'v', on a level road around a curved path of radius 'R'. The forces acting on the car are;
(i) It weight 'mg' acting vertically downward.
(ii) Normal reaction N acting vertically upward.
(iii) A horizontal force $\vec{F}$ on the car that provides necessary centripital force to keep the car in circular path. This force is provided by the friction b/w the tyres of car & road.



However these forces are not large enough all the times to rely upon. So, for a safe turn around the curved path, the road must be banked. When the road bed is banked

at an angle θ. Then the normal reaction 'N' has two rectangular components $N\cos\theta$ and $N\sin\theta$.
As there is no motion along the mg and '$N\cos\theta$', therefore they cancel away each other.

$$\therefore \boxed{N\cos\theta = mg} \quad \text{——— (1)}$$

The horizontal or radial component of N is '$-N\sin\theta$', which provides the necessary centripital force

$$-N\sin\theta = ma_R = -\frac{mv^2}{R}$$

or

$$\boxed{N\sin\theta = mv^2/R} \quad \text{——— (2)}$$

Dividing equ. (2) by equ. (1), we get

$$\frac{N\sin\theta}{N\cos\theta} = \frac{mv^2}{R} \times \frac{1}{mg}$$

$$\tan\theta = \frac{v^2}{Rg}$$

It shows that angle of banking 'θ' depends on the speed of car, and radius of curvature of the path and not on the mass of car.

×————×

Equations of Motion (Constant Forces)

Forces are said to be constant if they donot depend on time, velocity or position. If a force is constant than acceleration is constant.

Suppose a constant force acts on a body and produces an acceleration 'a' in it. then by definition;

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

Integrating both the sides, we get

$$\int dv = \int a\,dt$$

On L.H.S limits are from v_0 to v & on R.H.S from 0 to t.

$$\int_{v_0}^{v} dv = \int_0^t a\,dt$$

$$\int_{v_0}^{v} dv = a\int_0^t dt$$

$$|v|_{v_0}^{v} = a|t|_0^t$$

$$v - v_0 = at$$

or $$v = v_0 + at$$

$$\boxed{v(t) = v_0 + at}$$ This equ. gives velocity at any time 't'.

equ. (1)

Now

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$dx = (v_0 + at)\,dt$$

$$dx = v_0 dt + at\,dt$$

Integrating $$\int dx = \int v_0 dt + \int at\,dt$$

Suppose the body is at a position x_0 at $t=0$ and at position x at time

$$\therefore \int_{x_0}^{x} dx = v_0\int_0^t dt + a\int_0^t t\,dt$$

$$|x|_{x_0}^{x} = v_0|t|_0^t + a\left|\frac{t^2}{2}\right|_0^t$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

or $$\boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2}$$ —— (2)

This equ. gives the position of body at any time 't'.

Examples of constant forces are (i) gravity near the earth's surface (ii) friction forces (iii) Tension in a string.

Non Constant Forces:

Following are the types of non-constant forces.

(i) Time dependent Forces:

When a force varies with time, it is called time dependent forces.

When brakes are applied to a running car, we first apply brakes slowly at high speed and then more strongly at slow speed. So braking force is time dependent. Time dependent force is an example of non-constant force.

Another example of time dependent force is provided by a travelling wave in a medium. e.g sound wave which varies sinunoidally with time.

(ii) Velocity - dependent Forces:

The force which changes with velocity of the body is called velocity dependent force. e.g. drag force experienced by a body moving through various medium such as air, water. The friction force increases with velocity.

(iii) Position - dependent Forces:

The forces which vary with position are called position dependent forces. e.g gravitational force, coulomb's force and elastic restoring force exerted by a spring stretched a distance x from mean ~~position~~

Analytical Methods for time - dependent Forces

For a time dependent force in one dimension, we can write

$$a(t) = \frac{dV(t)}{dt}$$

$$dV(t) = a(t)\,dt.$$

Integrating both sides b/w the limits $V(t) = V_0$ at $t=0$ & $V(t) = V$ at time 't'.

$$\therefore \int_{V_0}^{V} dV(t) = \int_0^t a(t)\,dt$$

$$|V|_{V_0}^{V} = \int_0^t a(t)\,dt$$

$$V - V_0 = \int_0^t a(t)\,dt$$

$$\boxed{V = V_0 + \int_0^t a(t)\,dt}$$

This is the same equation as $V = V_0 + at$ with only difference that 'a' is inside the integral sign.

Also

$$V(t) = \frac{dx(t)}{dt}$$

$$dx(t) = V(t)\,dt$$

Integrating both sides b/w the limits $x(t) = x_0$ when $t=0$ & $x(t) = x$

at time 't'

$$\int_{x_0}^{x} dx(t) = \int_0^t V(t)\,dt$$

$$|x|_{x_0}^{x} = \int_0^t V(t)\,dt$$

$$x - x_0 = \int_0^t V(t)\,dt$$

$$\boxed{x(t) = x_0 + \int_0^t V(t)\,dt}$$

Sample Problem-4

Sample Problem 4 A car is moving at 105 km/h (about 65 mi/h or 29.2 m/s). The driver suddenly begins to apply the brakes, but does so with increasing force so that the deceleration increases with time according to $a(t) = ct$, where $c = -2.67$ m/s³. (*a*) How much time passes before the car comes to rest? (*b*) How far does it travel in the process?

Solution:

$V_0 = 105\ km/h = 29.2\ m/s$

$V = 0$

$C = -2.6\ m/s^2$

(a) $t = ?$ (b) $x = ?$

As: $$V = V_0 + \int_0^t a(t)\,dt$$

Putting $a(t) = ct$ $$V = V_0 + \int_0^t ct\,dt$$

$$V = V_0 + c\frac{t^2}{2}$$

$$V = V_0 + \frac{1}{2}ct^2 \quad \text{——— (1)}$$

Put $V = 0$

$$0 = V_0 + \frac{1}{2}ct^2$$

$$-V_0 = \frac{1}{2}ct^2$$

$$t^2 = \frac{-2V_0}{c}$$

$$t^2 = \frac{-2 \times 29.2}{-2.67}$$

$$t^2 = 21.87$$

$$t = \sqrt{21.87}$$

$$\boxed{t = 4.68\ sec.}$$ Ans-I

(b) For $x = ?$

$$x = x_0 + \int_0^t V(t)\,dt$$

$$x = x_0 + \int_0^t \left(V_0 + \frac{1}{2}ct^2\right)dt$$

From equ (1)
$V(t) = V_0 + \frac{1}{2}ct^2$

$$x = x_0 + \int_0^t V_0 \, dt + \frac{1}{2} c \int_0^t t^2 dt$$

$$x = x_0 + V_0 t + \frac{1}{2} c \left(\frac{t^3}{3}\right)$$

$$x = \cancel{x_0 +} V_0 t + \frac{1}{5} c t^3$$

Putting $x_0 = 0$
$t = 4.68$ sec.

$$x = 0 + 29.2 \times 4.68 + \frac{1}{6}(-2.67)(4.68)^3$$

$$x = 136.656 + (-45.61)$$

$$\boxed{x = 91.00 \text{ m}}$$

Effect of Drag Forces on Motion

Consider an object of mass 'm' falling in air. It experiences a drag force 'D', which increases linearly with velocity. i.e

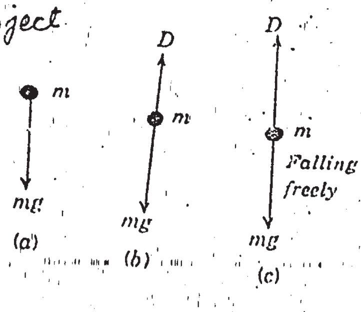
$$D \propto V$$

$$D = bV \quad \text{where } b \text{ is constt.}$$

'b' (constant) depends upon the properties of object & properties of fluid i.e

(i) Shape and size of object

(ii) Density of fluid.



The drag force acts opposite to velocity. When the object is released, 'D' is zero as 'V' is zero, at start and 'D' increases as v increases.

At a certain stage, the value of drag force become equal to weight of the object. i.e $D = mg$, then the net force on the object is zero and object falls with maximum and uniform velocity called terminal velocity.

By IInd Law of motion, along y-axis we have

$$\sum F_y = ma$$

or

$$mg - D = ma$$

$$mg - bV = ma$$

$$a = g - \frac{bV}{m}$$

Now when the body moves with uniform velocity, acceleration becomes zero and $V = V_T$

$$\therefore \quad 0 = g - \frac{b}{m} V_T$$

$$g = \frac{b}{m} V_T$$

$$\boxed{V_T = \frac{mg}{b}} \quad \text{——— (1)}$$

When the force and acceleration are function of velocity then acceleration is given by;

$$a(v) = \frac{dv}{dt}$$

$$\frac{dv}{a(v)} = dt \quad \text{——— (2)}$$

For downward motion, net force in downward direction is given as:

$$mg - bv = ma$$

$$a = g - \frac{bv}{m}$$

Putting this value of "a" in equ (2), we get

$$\frac{dv}{g - \frac{bv}{m}} = dt$$

Integrating both sides between limits $V=0$ at $t=0$ and $V=V$ at time "t", we get

$$\int_0^V \frac{dv}{g - \frac{bv}{m}} = \int_0^t dt$$

$$\int_0^V \frac{dv}{\frac{mg - bv}{m}} = |t|_0^t$$

$$m \int_0^V \frac{dv}{mg - bv} = t - 0$$

$$-\frac{m}{b} \int_0^V \frac{-b\,dv}{mg - bv} = t$$

or $$-\frac{m}{b} \left| \log(mg - bv) \right|_0^V = t$$

or $\quad -\frac{m}{b}\log(mg - bv) \quad \frac{m}{b}\log mg = t$

$$-\frac{m}{b}\left[\log(mg - bv) - \log(mg)\right] = t$$

$$\log\left(\frac{mg - bv}{mg}\right) = -\frac{bt}{m}$$

$$\frac{mg - bv}{mg} = e^{-bt/m}$$

$$\frac{\cancel{mg}}{\cancel{mg}} - \frac{bv}{mg} = e^{-bt/m}$$

$$1 - \frac{bv}{mg} = e^{-bt/m}$$

$$\left(1 - e^{-bt/m}\right) = \frac{bv}{mg}$$

or $\quad V = \frac{mg}{b}\left(1 - e^{\frac{-bt}{m}}\right)$

$$V(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right) \quad \text{---} \quad 3$$

In the start when 't' is small

$$e^{x} = 1 + x$$

$$e^{x} = 1 - x$$

So equ. 3 becomes

$$V(t) = \frac{mg}{b}\left(1 - \left(1 - \frac{bt}{m}\right)\right)$$

$$V(t) = \frac{mg}{b}\left(1 - 1 + \frac{bt}{m}\right)$$

$$V(t) = \frac{\cancel{m}g}{\cancel{b}}\left(\frac{\cancel{b}t}{\cancel{m}}\right)$$

$$\boxed{V(t) = gt}$$

Before the drag force become significant, the object fall freely with acceleration $a = g$. But after a long time when $t \to \infty$

$$e^{-\infty} \to 0$$

Equ. 3 becomes $\quad V(t) = \frac{mg}{b}(1 - 0)$

$V(t) = \frac{mg}{b}$ which is the terminal velocity of the body.

$$\boxed{V_T = \frac{mg}{b}}$$

Problem:

Find expressions for (i) acceleration (ii) Displacement for small and large 't' from the expression
$V(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$

Solution:

(i) For acceleration

$$a(t) = \frac{dV(t)}{dt}$$

$$a(t) = \frac{mg}{b}\left[0 + \frac{b}{m} e^{-bt/m}\right]$$

$$a(t) = \frac{\cancel{m}g}{\cancel{b}}\left[\frac{\cancel{b}}{\cancel{m}} e^{-bt/m}\right]$$

$$\boxed{a(t) = g e^{-bt/m}} \quad (1)$$

This is the expression for acceleration. For small 't' i.e. t=0

$$a = ge^{-0} = \frac{g}{e^0} = g \qquad \because e^0 = 1$$

$$\boxed{a = g}$$

For large t; $e^{-bt/m} = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$

$$\boxed{a = 0}$$

(ii) For displacement

$$\frac{dx}{dt} = V(t)$$

$$\therefore \frac{dx}{dt} = \frac{mg}{b}\left[1 - e^{-bt/m}\right]$$

$$\int_0^x \frac{dx}{dt}\,dt = \int_0^t \frac{mg}{b}\left[1 - e^{-bt/m}\right] dt$$

$$x(t) = \frac{mg}{b}\int_0^t \left(1 - e^{-bt/m}\right) dt$$

$$= \frac{mg}{b}\left[t - \frac{e^{-bt/m}}{-b/m}\right]_0^t$$

$$= \frac{mg}{b}\left[t + \frac{m}{b} e^{-bt/m}\right]_0^t$$

$$= \frac{mg}{b}\left[\left(t + \frac{m}{b} e^{-bt/m}\right) - \left(0 + \frac{m}{b}\right)\right]$$

$$x(t) = \frac{mg}{b}\left[t + \frac{m}{b} e^{-bt/m} - \frac{m}{b}\right] \quad \text{---} \quad (2)$$

This is the expression for displacement
For small 't' i.e at $t=0$

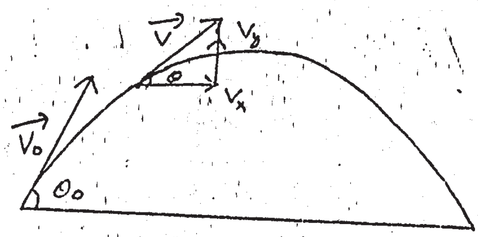
$$x = \left[0 + \frac{m}{b} - \frac{m}{b}\right]$$

$$\boxed{x = 0}$$

Projectile Motion with air resistance:

A body is projected at some angle θ above or below the horizontal is called a projectile and its its motion is called projectile motion.

If there were no gravity, the body will continue to move in a straight line with the same initial velocity according to the first law of motion. However due to gravity, it moves along a parabolic path as shown. So projectile motion is an example of motion with uniform accelerations in a plane.



Consider a body of mass 'm' projected at an angle θ_0 to the horizontal in x-y-plane with velocity $\vec{V_0}$. The frictional force D is opposite to the velocity $\vec{V}$.

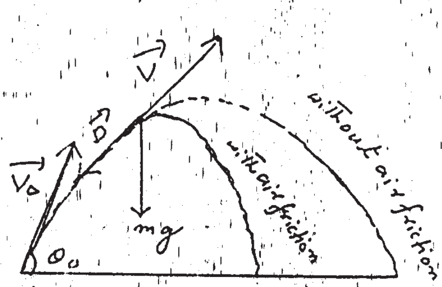
∴ This frictional force is given by;

$$D = -bV$$

In the presence of friction

(i) Range of projectile is decreased &

(ii) Max. height is also reduced.



So the trajectory does not remain a true parabola. The trajectory is not symmetric about the maximum. The descending motion is much steeper than the ascending motion.

The drag force depends on the velocity of projectile in still air. If the wind is blowing, the result will be different.

Frame of Refrence:

It is the place or surrounding according to which we take some observation, e.g. Laboratory, moon etc. Frame of refrence are of two types.

(i) Inertial frame of refrence

(ii) Non inertial frame of refrence.

Inertial Frame

It is that fram whose acceleration is zero. Newton's laws of motion are correct only in this frame. Pure inertial frame does not exist in the nature.

Non-Inertial Frame

It is an accelerated frame of refrence. Newton's laws of motion are not correct in this frame.

Non-inertial frames & Pseudo Forces:

According to newtons IInd law of motion the force acting on the body is obtained by product of mass and acceleration of the body.

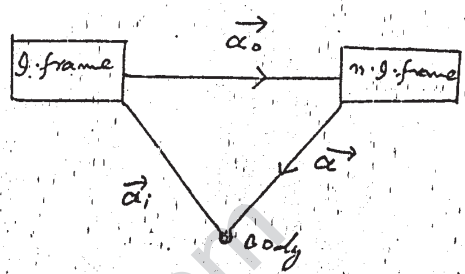
i.e $\vec{F} = m\vec{a}$

This law is true only in an inertial frame because the inertial frame has no acceleration.

Let us see what happens when the observer is in non intertial frame.

Suppose acceleration of the non inertial frame is $\vec{a}_0$ w.r.t. inertial frame.

Let $\vec{a}_i$ be the acceleration of the body w.r.t the inertial frame and $\vec{a}$ is the acceleration of the body as found by the observer in non inertial frame.



The force found by the observer in non-inertial frame is

$$\vec{F} = m\vec{a}$$

But this is not correct because observer does not account for his own acceleration.

The true force on the body is;

$$\vec{F} = m\vec{a}_i$$

Thus the true force in non inertial frame can be obtained only if we consider the acceleration of non-inertial frame also. Thus

$$\vec{a}_i = \vec{a} + \vec{a}_0$$

So force on body is

$$\vec{F} = m(\vec{a} + \vec{a}_0)$$

$$\vec{F} = m\vec{a} + ma_0$$

$$\vec{F} - m\vec{a}_0 = m\vec{a}$$

Putting $-m\vec{a}_0 = \vec{F}_0$ $\qquad \vec{F} + \vec{F}_0 = m\vec{a}$

The quantity $\vec{F}_0 = -m\vec{a}_0$ is called Pseudo force or fictitious force. It simply represents the effect of acceleration of non-inertial frame. Fictitious force is not applied by any external agency but it is felt by the observer in non-inertial frame due to acceleration of non inertial frame.

Examples of Fictitious Forces:

(a) A freely falling body or Elevator

Consider a freely falling elevator as a non-inertial frame of reference. Its acceleration is;

$$\vec{a}_o = -g\hat{z}$$

The pseudo force on the body of mass 'm' in this frame is;

$$\vec{F}_o = -m\vec{a}_o$$

$$= -m(-g\hat{z})$$

$$\boxed{\vec{F}_o = mg\hat{z}}$$

A body in the elevator is acted upon by two forces;

1. Pseudo force $\vec{F}_o = mg\hat{z}$
2. force of gravity $\vec{F} = -mg\hat{z}$

$\therefore$ Total apparent force on the body in non-inertial frame is equal to $mg\hat{z} + (-mg\hat{z}) = 0$

$$\therefore \boxed{\vec{F} + \vec{F}_o = 0}$$

So the body is unaccelerated in non-inertial frame. This is the state of weightlessness.

(b) Fictitious Forces in a rotating System:

Suppose a body of mass 'm' at rest on smooth surface of table. If the table is at rest then it is an inertial frame, and so

$$\vec{F} = m\vec{a}_i \qquad \because a_i = 0$$

$$\boxed{\therefore \vec{F} = 0}$$

Now suppose the table begins to rotate with uniform angular velocity 'ω' about the vertical axis. If the body is at a distance r from the axis of rotation the centripetal acceleration is

$$\vec{a}_o = -\vec{r}\omega^2 \quad \text{directed towards axis of rotation.}$$

The pseudo force $\vec{F}_o = -m\vec{a}_o = -m(-\vec{r}\omega^2)$

$\vec{F}_o = m\vec{r}\omega^2$, away from axis of rotation

It is called centrifugal force.

Weightlessness in Satellite:

We know that a body in a satellite is weightless. So the body's apparent weight is zero, inside the satellite.
The weightlessness is due to appearance of fictitious force which is equal and opposite to the real weight of the body.

x————x

Coriolis Forces

It is another example of pseudo force which appears as a result of daily rotation of earth. The rotating earth forms a non-inertial frame.
Due to coriolis forces on a stone dropped from a high tower seem to strike the ground a little to the east of the vertical.

Coriols force also place a role in the motion of atmosphere around the centre of low or high pressures. This coriols effect is responsible for the circulation of wind in a cyclon. Coriols force of rotating earth also effects the paths of long range shells.

x————x

The End