Nasir Pervaiz Butt
M. Sc. (Physics) M. Phil
Assistant Professor
Govt. College Sargodha.
House No. 338 St. No. 7
Cheema Colony,
Sargodha.

Chapter-6

COLLISIONS

Collisions:

A collision is said to occure between two bodies if their velocities are changed after they strike each other.

Compression Time:

The time for which two bodies remain in contact is called compression time. This time is smaller as compared to the time of observation.

Direct or Head-on Collision:

If the relative velocities of the two colliding bodies at the time of collision are along the ~~normal~~ common normal at the point of contact, the collision is said to be direct or head-on.

Oblique Collision:

If the relative velocities of two colliding bodies at the time of collision are not along the common normal at the ~~time of~~ point of contact, the collision is said to be oblique.

Elastic Collision:

It is that collision in which total momentum and total K.E of the system remain conserved before and after the collision. e.g collision b/w two rigid bodies i.e billiard balls is elastic.

Inelastic Collision:

It is that collision in which total momentum of the system remains conserved but total energy does not remain conserved.

Impulsive Forces:

The forces which act for a short time

as compared to the time of observation of the system are called impulsive forces.

x———————x

1. Conservation of Momentum During Collision

Consider two particles having masses m_1 and m_2. Suppose a collision takes place b/w them. During the collision the two particles exert impulsive forces on each other.

Let $\vec{F}_{12}$ be the force exerted by m_2 on m_1, and $\vec{F}_{21}$ is the force exerted by m_1 on m_2.

Collision between two particles

By Newton's 3rd law of motion

$$\vec{F}_{12} = -\vec{F}_{21}$$

The change in momentum of particle of mass m_1 is given by

$$\Delta\vec{P}_1 = \int_{t_i}^{t_f} \vec{F}_{12}\, dt = \langle\vec{F}_{12}\rangle \Big| t \Big|_{t_i}^{t_f} = \langle\vec{F}_{12}\rangle (t_f - t_i)$$

$$\Delta\vec{P}_1 = \langle\vec{F}_{12}\rangle \Delta t \quad \text{———} \quad (1)$$

Where $\langle\vec{F}_{12}\rangle$ is the average force during the time $\Delta t = t_f - t_i$;

The change in momentum of the particle of mass m_2 during the collision is given by

$$\Delta\vec{P}_2 = \int_{t_i}^{t_f} \vec{F}_{21}\, dt = \langle F_{21}\rangle \Big| t \Big|_{t_i}^{t_f} = \langle\vec{F}_{21}\rangle (t_f - t_i)$$

$$\Delta\vec{P}_2 = \langle\vec{F}_{21}\rangle \Delta t \quad \text{———} \quad (2)$$

where $\langle\vec{F}_{21}\rangle$ is the average value of force $\vec{F}_{21}$ during the time $\Delta t = t_f - t_i$

If no other force acts on the particles, then

$$\vec{F}_{12} = -\vec{F}_{21}$$

and $\langle \vec{F}_{12} \rangle = -\langle \vec{F}_{21} \rangle$

$$\Delta\vec{P}_1 = -\Delta\vec{P}_2$$

$$\Delta\vec{P}_1 + \Delta\vec{P}_2 = 0$$

If the two particles form isolated system, then the total momentum of the system

$$\vec{P} = \vec{P}_1 + \vec{P}_2$$

So the total change in momentum of the system is zero; i.e

$$\Delta\vec{P} = \Delta\vec{P}_1 + \Delta\vec{P}_2 = 0$$

Hence we conclude that in the absence of external forces, the total momentum of the two particle system is not changed during the collision. This is called the "Law of Conservation Of Momentum," for two particles system.

The impulsive force acting during the collision are internal forces and have no effect on the total momentum of the system.

2- Elastic Collision in One Dimension

Collision is said to be in one dimension if colliding bodies move along the same straight line before and after the collision.

Consider two non-rotating spheres having masses m_1 and m_2 moving on a smooth horizontal surface along a straight line joining their centres. Suppose V_1 and V_2 are the velocities of m_1 and m_2 respectively before collision. Let $V_1 > V_2$. The spheres collide and their velocities become V_1' and V_2' after the collision.

After the collision they move along the same straight line joining their centres. As the collision is elastic, so we can apply

(i) Law of Conservation of Momentum

(ii) Law of ~~conservation~~ of Kinetic Energy.

By the law of conservation of momentum;

Total initial momentum = Total final momentum

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$$m_1V_1 - m_1V_1' = m_2V_2' - m_2V_2$$

$$m_1(V_1 - V_1') = m_2(V_2' - V_2) \text{ ———— } (1)$$

By the law of conservation of kinetic energy;

Total initial kinetic energy = Total Final K.E.

$$\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2$$

$$m_1V_1^2 + m_2V_2^2 = m_1V_1'^2 + m_2V_2'^2$$

$$m_1V_1^2 - m_1V_1'^2 = m_2V_2'^2 - m_2V_2^2$$

$$m_1(V_1^2 - V_1'^2) = m_2(V_2'^2 - V_2^2)$$

$$m_1(V_1 - V_1')(V_1 + V_1') = m_2(V_2' - V_2)(V_2' + V_2) \text{ ———— } (2)$$

Dividing (2) by (1) we get

$$\frac{m_1(V_1 - V_1')(V_1 + V_1')}{m_1(V_1 - V_1')} = \frac{m_2(V_2' - V_2)(V_2' + V_2)}{m_2(V_2' - V_2)}$$

$$(V_1 + V_1') = V_2' + V_2$$

$$V_1 - V_2 = V_2' - V_1'$$

$$V_1 - V_2 = -(V_1' - V_2') \quad \text{--- (3)}$$

$V_1 - V_2$ is the relative velocity of approach before collision and $V_1' - V_2'$ is relative velocity of separation after collision.

So we find that relative velocity of approach before collision is equal and opposite to the relative velocity of separation after collision.

To find V_1' and V_2', From equation (3)

$$V_2' = V_1 - V_2 + V_1'$$

By putting the value of V_2' in equ. (1), we get

$$m_1(V_1 - V_1') = m_2(V_2' - V_2) \quad \text{--- (1)}$$

$$m_1(V_1 - V_1') = m_2(V_1 - V_2 + V_1' - V_2)$$

$$m_1(V_1 - V_1') = m_2(V_1 + V_1' - 2V_2)$$

$$m_1V_1 - m_1V_1' = m_2V_1 + m_2V_1' - 2m_2V_2$$

$$m_1V_1 - m_2V_1 = m_2V_1' + m_1V_1' - 2m_2V_2$$

$$(m_1 - m_2)V_1 + 2m_2V_2 = (m_1 + m_2)V_1'$$

$$V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1 + \frac{2m_2}{m_1 + m_2}V_2 \quad \text{--- (4)}$$

From equ. (3),

$$V_1' = V_2' - V_1 + V_2$$

By putting the value of V_1' in equ. (1) we get

$$m_1(V_1 - V_1') = m_2(V_2' - V_2)$$

$$m_1(V_1 - V_2' + V_1 - V_2) = m_2(V_2' - V_2)$$

$$m_1(2V_1 - V_2' - V_2) = m_2(V_2' - V_2)$$

$$2m_1V_1 - m_1V_2' - m_1V_2 = m_2V_2' - m_2V_2$$

$$2m_1V_1 + m_2V_2 - m_1V_2 = mV_2' + m_2V_2'$$

$$(2m_1)V_1 + (m_2-m_1)V_2 = (m_1+m_2)V_2'$$

$$\therefore \quad V_2' = \frac{m_2-m_1}{m_1+m_2}V_2 + \frac{2m_1}{m_1+m_2}V_1 \quad \text{------} \quad (5)$$

Equ. (4) and equ. (5) hold in all inertial reference frames and give the final velocities after collision.

Special Cases

(i) Equal Masses:

When $m_1 = m_2$ then from the above equation (4) and equation (5), we get

$$V_1' = V_2 \quad \text{and} \quad V_2' = V_1$$

i.e the particles interchange their velocities after collision.

(ii) Target Particle at rest (i.e $V_2 = 0$)

If $V_2 = 0$, then from equ. (4) and equ. (5) we have

$$V_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right)V_1 \quad \text{and} \quad V_2' = \left(\frac{2m_1}{m_1+m_2}\right)V_1$$

If we put $m_1 = m_2$ in this case, then

$$V_1' = 0 \quad \text{and} \quad V_2' = V_1$$

i.e the first particle stops and the second particle moves with the velocity of the first.

(iii) Massive Target: (i.e $m_2 >> m_1$)

If $m_2 >> m_1$ then from equ. (4) & equ. (5), we have

$$V_1' = -V_1 + 2V_2 \qquad V_2' = V_2$$

If the target particle is at rest (i.e $V_2 = 0$) then

$$V_1' = -V_1 \quad \text{and} \quad V_2' = 0$$

Hence when a light particle collides with a heavy target particle at rest then the light particle rebounds with the same speed while target particle remains at rest

(iv) **Massive Particle**: (i.e $m_1 >> m_2$)

If $m_1 >> m_2$ then from equ: ④ and equ ⑤ we get;

$$V_1' = V_1 \qquad \text{and} \qquad V_2' = 2V_1 - V_2$$

If the target particle is at rest (i.e $V_2 = 0$) then

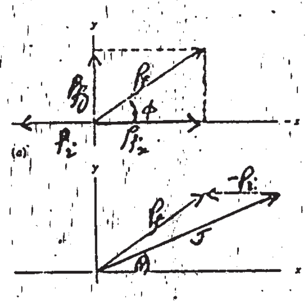
$$V_1' = V_1 \qquad \text{and} \qquad V_2' = 2V_1$$

So when a heavy particle collides with a light par- ~~ti~~ -ticle at rest then the first particle goes on moving with the same speed and the target particle starts moving with a velocity double than the velocity of the first.

x———————x

Sample Problem - 1

Sample Problem 1 A baseball (which has an official weight of about 5 oz) is moving horizontally at a speed of 93 mi/h (about 150 km/h) when it is struck by the bat (see Fig. 1). It leaves the bat in a direction at an angle $\phi = 35°$ above its incident path and with a speed of 180 km/h. (*a*) Find the impulse of the force exerted on the ball. (*b*) Assuming the collision lasts for 1.5 ms (= 0.0015 s), what is the average force? (*c*) Find the change in the momentum of the bat.



Sol:

Weight of the base ball = 5 oz

mass " " " " $m = 0.14$ kg

Initial speed of base ball $V_i = 150\ \text{km/h} = \frac{150 \times 1000}{3600} = 42\ \text{m/s}$

Angle $\phi = 35°$

Final speed of the base ball $V_f = 180\ \text{km/h} = \frac{180 \times 1000}{3600} = 50\ \text{m/s}$

(a) Impulse of the force exerted on the ball $J = ?$

Impulse can be calculated as follows;

$$P_{fx} = P_f \cos\phi = m V_f \cos\phi$$

$$= 0.14 \times 50 \times \cos 35°$$

$$= 7 \times 0.819$$

$$= 5.7\ \text{kg m/s} \qquad \text{(Final momentum along x-axis)}$$

Final momentum of the ball along y-axis is given as;

$$P_{fy} = P_f \sin\phi = mV_f \sin\phi$$
$$= 0.14 \times 50 \times \sin 35^\circ$$
$$= 7 \times 0.573$$
$$= 4.0 \text{ kg m/sec.}$$

Now initial momentum of the ball along x-axis is;

$$P_{ix} = P_i = -mV_i \cos\phi$$
$$= -0.14 \times 42 \times \cos 0^\circ$$
$$= -5.9 \times 1 \qquad \because \cos 0^\circ = 1$$
$$P_{ix} = -5.9 \text{ kg m/s}$$

Where -ve sign stands for momentum along -ve x-axis.

Now the x-component of impulse is;

$$J_x = P_{fx} - P_{ix}$$
$$J_x = 5.7 - (-5.9)$$
$$J_x = 11.6 \text{ kg m/s}$$

& y-component of impulse is;

$$J_y = P_{fy} - P_{iy}$$
$$J_y = 4 - 0 \qquad \because P_{iy} = 0 \text{ because ball is moving along x-axis}$$
$$J_y = 4 \text{ kg m/s}$$

Now

$$J = J_x^2 + J_y^2$$
$$J = \sqrt{(J_x)^2 + (J_y)^2}$$
$$J = \sqrt{(11.6)^2 + (4)^2}$$
$$= \sqrt{134.56 + 16}$$
$$= \sqrt{150.56}$$
$$\boxed{J = 12.3 \text{ kg m/s}} \quad \text{Ans.}$$

Now for direction

$$\tan\theta = \frac{J_y}{J_x}$$
$$\theta = \tan^{-1}(J_y/J_x)$$
$$= \tan^{-1}(4/11.6)$$
$$\boxed{\theta = 19^\circ} \quad \text{Ans}$$

Magnitude of impulse $J = 12.3 \, kg\,m/s$
Direction of impulse $\theta = 19^\circ$ Ans.

(b) Average Force $F = ?$

$\Delta t = 0.0015$ sec.

As we know that

$$F \Delta t = J$$

$$F = J/\Delta t$$

$$F = 12.3/0.0015$$

$$\boxed{F = 8200\ N}$$ Ans.

(c) Change in momentum of the bat $= ?$

As we know that

Chang in momentum $\Delta P = J$

$\Delta P = 12.3 \, kg\,m/s$ Ans.

x———————x

Sample Problem. 2

Sample Problem 2 (*a*) By what fraction is the kinetic energy of a neutron (mass m_1) decreased in a head-on elastic collision with an atomic nucleus (mass m_2) initially at rest? (*b*) Find the fractional decrease in the kinetic energy of a neutron when it collides in this way with a lead nucleus, a carbon nucleus, and a hydrogen nucleus. The ratio of nuclear mass to neutron mass ($= m_2/m_1$) is 206 for lead, 12 for carbon, and 1 for hydrogen.

Solution:

Mass of neutron $= m_1$

Mass of atomic nucleus $= m_2$; $v_2 = 0$

By what fraction the K·E of a neutron decreased $= ?$

The fractional decrease in K·E of neutron is

$$\frac{K_i - K_f}{K_i} = \frac{\frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_1 v_1'^2}{\frac{1}{2} m_1 v_1^2}$$

$$= \frac{v_1^2 - v_1'^2}{v_1^2}$$

$$= \frac{\cancel{v_1^2}\left(1 - v_1'^2/v_1^2\right)}{\cancel{v_1^2}}$$

$$\frac{K_i - K_f}{K_i} = 1 - \frac{V_1'^2}{V_1^2} \text{ ——— } (1)$$

Now we know that velocity of neutron after collision with mass target at rest is given by

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1$$

$$\frac{V_1'}{V_1} = \frac{m_1 - m_2}{m_1 + m_2}$$

By putting $\frac{V_1'}{V_1}$ in equ. (1) we get

$$\frac{K_i - K_f}{K_i} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

$$= 1 - \left(\frac{m_1^2 + m_2^2 - 2m_1m_2}{(m_1 + m_2)^2}\right)$$

$$= 1 - \frac{(m_1^2 + m_2^2 - 2m_1m_2)}{(m_1 + m_2)^2}$$

$$= \frac{(m_1 + m_2)^2 - (m_1^2 + m_2^2 - 2m_1m_2)}{(m_1 + m_2)^2}$$

$$= \frac{\cancel{m_1^2} + \cancel{m_2^2} + 2m_1m_2 - \cancel{m_1^2} - \cancel{m_2^2} + 2m_1m_2}{(m_1 + m_2)^2}$$

$$\boxed{\frac{K_i - K_f}{K_i} = \frac{4m_1m_2}{(m_1 + m_2)^2}} \quad \text{Ans.}$$

(b) Fractional decrease in K.E of neutron when it collides with ∴ Lead nucleus for which $\frac{m_2}{m_1} = 206$

$$\therefore m_2 = 206 m_1$$

$$\frac{K_i - K_f}{K_i} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

$$\frac{K_i - K_f}{K_i} = \frac{4m_1 (206 m_1)}{(m_1 + 206 m_1)^2}$$

$$= \frac{824 m_1^2}{(207 m_1)^2}$$

$$= \frac{824 m_1^2}{207 \times 207 m_1^2}$$

$$\frac{K_i - K_f}{K_i} = 0.019$$

$$\frac{K_i - K_f}{K_i} = 0.02$$

$$\boxed{\frac{K_i - K_f}{K_i} = 2\%}$$ Ans.

(ii) For carbon $\frac{m_2}{m_1} = 12$

$$m_2 = 12 m_1$$

$$\therefore \frac{K_i - K_f}{K_i} = \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

$$= \frac{4m_1 (12 m_1)}{(m_1 + 12 m_1)^2}$$

$$= \frac{48 m_1^2}{(13 m_1)^2}$$

$$= \frac{48 m_1^2}{169 m_1^2}$$

$$\frac{K_i - K_f}{K_i} = 0.28$$

$$\boxed{\frac{K_i - K_f}{K_i} = 28\%}$$ Ans.

(iii) For Hydrogen $\dfrac{m_2}{m_1} = 1$

$$m_2 = m_1$$

$$\frac{K_i - K_f}{K_i} = \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

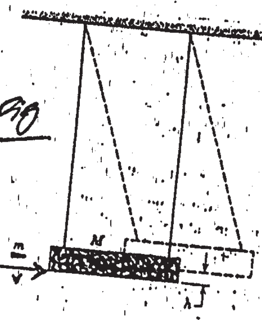
$$= \frac{4 m_1 m_1}{(m_1 + m_1)^2} \qquad \because m_2 = m_1$$

$$= \frac{4 m_1^2}{4 m_1^2} = 1$$

$$\boxed{\frac{K_i - K_f}{K_i} = 100\%} \quad \text{Ans.}$$

Sample Problem 3 A ballistic pendulum (Fig. 11) is a device that was used to measure the speeds of bullets before electronic timing devices were available. It consists of a large bl[illegible]k of wood of mass M, hanging from two long pairs of cords. A bullet of mass m is fired into the block and comes quickly to rest relative to the block. The block + bullet combination swings upward, its center of mass rising a vertical distance h before the pendulum comes momentarily to rest at the end of its arc. Take the mass of the block to be $M = 5.4$ kg and the mass of the bullet to be $m = 9$[illegible] What is the initial speed of the bullet if the block rises to a height of $h = 6.3$ cm? (b) What is the initial [illegible] energy of the bullet? How much of this energy remains as mechanical energy of the swinging pendulum?

Fig



Sol → (m) Mass of bullet = 9.5 g = 9.5×10^{-3} kg

(M) Mass of block = 5.4 kg

(h) Height through which the block rises = 6.3 cm = 6.3 [illegible]

(a) Initial speed of the bullet, $v = ?$

As the bullet lodges itself within the block after collision. So it is completely inelastic collision. So K.E is not conserved during collision. But it is conserved after the collision.

By law of conservation of momentum along x-axis

$$mv = (M + m)V \quad \text{———— (1)}$$

where 'v' is the velocity of bullet after collision an V is the velocity of block-bullet combination after collision

After the collision

K.E of the system at bottom = P.E of the system at top

$$\therefore \quad \frac{1}{2}(M+m)V^2 = (M+m)gh$$

$$V^2 = 2gh$$

$$V = \sqrt{2gh}$$

By putting 'V' in equation ①, we get

$$mv = (M+m)\sqrt{2gh}$$

$$v = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$

$$= \left(\frac{5.4 + 9.5\times10^{-3}}{9.5\times10^{-3}}\right)\sqrt{2\times9.8\times6.3\times10^{-2}}$$

$$= \left(\frac{5.4+0.0095}{0.0095}\right)\sqrt{19.6\times0.063}$$

$$= 568 \times 1.11$$

$$\boxed{v = 630 \text{ m/s}}$$ Ans.

(b) Initial K.E of the bullet = ?

Initial K.E of the bullet $= \frac{1}{2}mv^2$

$$= \frac{1}{2}\times0.0095\times(630)^2$$

$\boxed{\text{Initial K.E of the bullet} = 1885 \text{ Joul}}$ Ans.

(c) Mechanical Energy of swinging pendulum = P.E of the "

$$E = (M+m)gh$$

$$= (5.4+0.0095)\times9.8\times0.063$$

$$= 5.4\times9.8\times0.63$$

$$\boxed{E = 3.3 \text{ Joul}}$$ Ans.

%age of initial K.E converted into mechanical Energy

i.e $\frac{3.3}{1885} \times 100$

$= 0.175\ \%$

$= 2\ \%$

---

3. Elastic Collision in Two Dimension:

"Elastic collision b/w two bodies is said to be in two dimensions if the direction of the motion of two bodies is not along the line joining their centres."

In this case the two bodies will not have a head on collision but an oblique collision. After oblique collision the two bodies will move in direction other than their original directions of motion.

Let m_1 be the mass of incident body moving with velocity 'V_1' along x-axis and the target body of mass m_2 is assumed to be at rest.

The perpendicular distance b/w line of motion of m_1 and a parallel line through m_2 at rest is called the impact parameter 'b'.

If $b=0$, the collision will be head-on.

If $b \neq 0$, the collision will be oblique

Fig. shows an oblique collision b/w two nuclei through their mutual electrostatic force of repulsion.

If V_1' and V_2' are the respective velocities of m_1 and m_2 after collision making angles ϕ_1 and ϕ_2 with original direction of motion, then by the law of conservation of momentum along x-axis

$$P_{i_x} = P_{f_x}$$

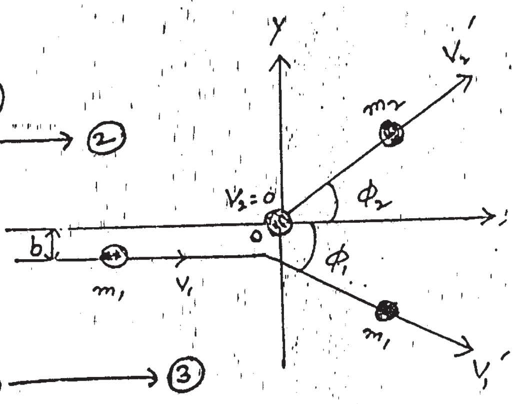
$$m_1 v_1 = m_1 v_1' \cos\varphi_1 + m_2 v_2' \cos\varphi_2 \quad \text{——— (1)}$$

Along y-axis

$$P_{i_y} = P_{f_y}$$

$$0 = -m_1 v_1' \sin\varphi_1 + m_2 v_2' \sin\varphi_2 \quad (\text{Equ. 2})$$

$$m_1 v_1' \sin\varphi_1 = m_2 v_2' \sin\varphi_2 \longrightarrow (2)$$



By the law of conservation of K.E, we get

$$(K\cdot E)_i = (K\cdot E)_f$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \quad (\text{equ. 3}) \longrightarrow (3)$$

As there are four unknown quantities v_1', v_2', φ_1, φ_2. So the three equations ~~cannot~~ equ. (1), equ. (2) and equ. (3) cannot determine them. So we must impose an additional condition to determine them.

For the purpose we suppose that φ_1 is known. Then v_1', v_2', φ_2 can be determined by equ. (1), (2) & (3) as explained in following sample problem-4.

Sample Problem-4

Sample Problem 4 A gas molecule having a speed of 322 m/s collides elastically with another molecule of the same mass which is initially at rest. After the collision the first molecule moves at an angle of 30° to its initial direction. Find the speed of each molecule after collision and the angle made with the incident direction by the recoiling target molecule.

Solution.

$v_1 = 322\, m/s$

$m_1 = m_2 = m$

$v_2 = 0$

$\phi_1 = 30°$

$v_1' = ?$ $\quad v_2' = ?$, $\quad \phi_2 = ?$

By the law of conservation of momentum;

$$m_1 v_1 = m_1 v_1' \cos\varphi_1 + m_2 v_2' \cos\varphi_2$$

$$m v_1 = m v_1' \cos\varphi_1 + m v_2' \cos\varphi_2$$

$$v_1 = v_1' \cos\varphi_1 + v_2' \cos\varphi_2 \quad \text{——— (1)}$$

and

$$0 = m_1 v_1' \sin\varphi_1 + m_2 v_2' \sin\varphi_2$$

$$m_1 v_1' \sin\varphi_1 = m_2 v_2' \sin\varphi_2$$

$$m v_1' \sin\varphi_1 = m v_2' \sin\varphi_2$$

$$v_1' \sin\varphi_1 = v_2' \sin\varphi_2 \quad \text{——— } (2)$$

From equ. (1)

$$v_1 - v_1' \sin\varphi_1 = v_2' \sin\varphi_2 \quad \text{——— } (3)$$

Squaring and adding equ. (2) and equ. (3), we get

$$(v_1' \sin\varphi_1)^2 + (v_1 - v_1' \cos\varphi_1)^2 = (v_2' \sin\varphi_2)^2 + (v_2' \cos\varphi_2)^2$$

$$v_1'^2 \sin^2\varphi_1 + v_1^2 + v_1'^2 \cos^2\varphi_1 - 2v_1 v_1' \cos\varphi_1 = v_2'^2 \sin^2\varphi_2 + v_2'^2 \cos^2\varphi_2$$

$$v_1'^2 (\sin^2\varphi_1 + \cos^2\varphi_1) + v_1^2 - 2v_1 v_1' \cos\varphi_1 = v_2'^2 (\sin^2\varphi_2 + \cos^2\varphi_2)$$

$$v_1'^2 + v_1^2 - 2v_1 v_1' \cos\varphi_1 = v_2'^2$$

$$v_1^2 + v_1'^2 - 2v_1 v_1' \cos\varphi_1 = v_2'^2 \quad \text{——— } (4)$$

By the law of conservation of K.E, we get

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$m v_1^2 = m_1 v_1'^2 + m v_2'^2$$

$$v_1^2 = v_1'^2 + v_2'^2 \quad \text{——— } (5)$$

By putting equ. (5) in equ. (4), we get

$$v_1'^2 + \cancel{v_2'^2} + v_1'^2 - 2v_1 v_1' \cos\varphi_1 = \cancel{v_2'^2}$$

$$2v_1'^2 - 2v_1 v_1' \cos\varphi_1 = 0$$

$$v_1'^2 - v_1 v_1' \cos\varphi_1 = 0$$

$$v_1'^2 = v_1 v_1' \cos\varphi_1$$

Dividing by v_1'

$$v_1' = v_1 \cos\varphi_1$$

$$v_1' = 322 \times \cos 30^\circ \qquad v_1 = 322\,m/s \quad \varphi_1 = 30^\circ$$

$$v_1' = 322 \times 0.866$$

$$\boxed{v_1' = 279\,m/s} \text{ Ans}$$

From equ. (5)

$$V_1^2 = V_1'^2 + V_2'^2$$

$$(322)^2 = (279)^2 + V_2'^2$$

Putting $V_1 = 322$
$V_1' = 279$ m

$$V_1'^2 = (322)^2 - (279)^2$$

$$V_1'^2 = 103684 - 77841$$

$$V_1'^2 = 25843$$

$$\boxed{V_1' = 161\ m/s}$$ Ans.

From equ. (2)

$$V_1' \sin\varphi_1 = V_2' \sin\varphi_2$$

By putting the values of V_1', V_2' & φ_1, we get

$$279 \times \sin 30^\circ = 161 \sin\varphi_2$$

$$\sin\varphi_2 = \frac{279}{161} \sin 30^\circ$$

$$= \frac{279}{161} \times 0.5$$

$$\sin\varphi_2 = 0.866$$

$$\varphi_2 = \sin^{-1}(0.866)$$

$$\boxed{\varphi_2 = 60^\circ}$$ Ans.

4. Inelastic Collision in one Dimension:

Consider an inelastic collision. By definition total momentum is conserved but total K.E is not conserved. It is to be noted that total energy is conserved.

Consider two particles of masses m_1 & m_2 moving with velocities v_1 and v_2 before collision. Suppose that the particles stick together after collision i.e the collision is totally inelastic. When two particles

~~stick~~ together, th ~~combination~~ moves with common velocity V'.
So by the law of conservation of momentum

Initial momentum = Final momentum

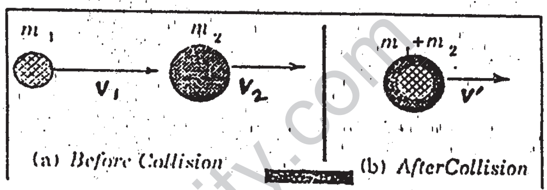
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) V'$$

$$V' = \left(\frac{m_1}{m_1+m_2}\right) v_1 + \left(\frac{m_2}{m_1+m_2}\right) V_2 \quad \text{—— (1)}$$

If m_2 is initially at rest i.e $V_2 = 0$ then

$$V' = \left(\frac{m_1}{m_1+m_2}\right) V_1 \quad \text{—— (2)}$$

Fig



It shows that greater is m_1, larger will be V'.
Equ. (1) can also be used in reverse, i.e a particle of mass M moving with velocity V' splits into two particles one of mass m_1 moving with velocity V_1 and the other of ~~velocity~~ mass $m_2 = (M - m_1)$ moving in the opposite direction with velocity V_2

By the law of conservation of momentum, we have

$$MV' = m_1 V_1 + m_2 V_2$$

If original particle is at rest then $V' = 0$ then

$$0 = m_1 V_1 + m_2 V_2$$

$$m_1 V_1 = -m_2 V_2$$

$$\boxed{\frac{V_1}{V_2} = \frac{-m_2}{m_1}}$$

i.e The product particles moves in opposite directions to conserve momentum.

5- Inelastic Collision in Two Dimensions:

Consider a completely inelastic collision in two dimensions, in which both the bodies are motion.

Suppose that mass m_1 is moving along x-axis with a velocity V_1. The body of mass m_2 is moving with velocity V_2 as shown. Suppose the two bodies stick together at origin and the total mass of the final body is $M = m_1 + m_2$

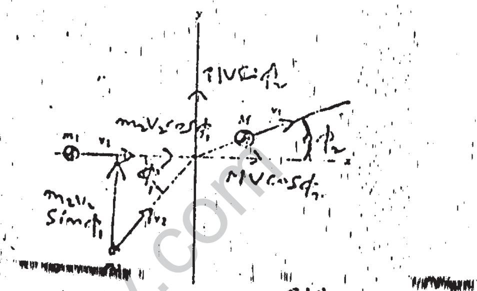


Fig 14

Suppose M moves with velocity V at angle φ_2 with x-axis

By the law of conservation of momentum, we have

$$m_1 V_1 + m_2 V_2 \cos\varphi_1 = MV \cos\varphi_2 \quad \text{———} \ (1)$$

Along y-axis

$$m_2 V_2 \sin\varphi_1 = MV \sin\varphi_2 \quad \text{———} \ (2)$$

Where $M = m_1 + m_2$ i.e total mass of the combination after the collision. The two unknown quantities V and φ_2 can be calculated by ~~solving~~ solving eq. (1) and (2)

x ——————————— x

Sample Problem: 5

Sample Problem 5 Two skaters collide and embrace in a completely inelastic collision. That is, they stick together after impact, as Fig. 15 suggests. Alfred, whose mass m_A is 83 kg, is originally moving east with a speed $v_A = 6.4$ km/h. Barbara, whose mass m_B is 55 kg, is originally moving north with a speed $v_B = 8.8$ km/h. (a) What is the velocity V of the couple after impact? (b) What is the fractional change in the kinetic energy of the skaters because of the collision?

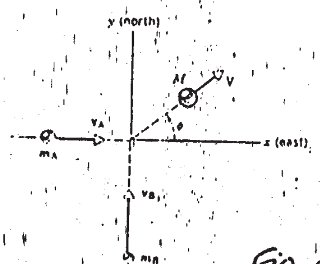


Fig 15

Solution.

$M_A = 83$ kg $\qquad M_B = 55$ kg

$V_A = 6.4$ km/h $\qquad V_B = 8.8$ km/h

(a) $V = ?$

By the law of conservation of momentum

$$P_{i_x} = P_{f_x}$$

$$m_A V_A = MV\cos\varphi \quad \text{——} (1)$$

and

$$P_{i_y} = P_{f_y}$$

$$m_B V_B = MV\sin\varphi \quad \text{——} (2)$$

Dividing equ. (2) by equ. (1), we get

$$\frac{m_B V_B}{m_A V_A} = \frac{MV\sin\varphi}{MV\cos\varphi}$$

$$\frac{m_B V_B}{m_A V_A} = \tan\varphi$$

$$\text{or} \quad \tan\varphi = \frac{m_B V_B}{m_A V_A}$$

$$\tan\varphi = \frac{55 \times 8.8}{83 \times 6.4}$$

$$\tan\varphi = 0.911$$

$$\varphi = \tan^{-1}(0.911)$$

$$\boxed{\varphi = 42.3^\circ}$$

From equ. (1)

$$m_A V_A = MV\cos\varphi$$

$$V = \frac{m_A V_A}{M\cos\varphi}$$

$$= \frac{83 \times 6.4}{138 \times \cos 42.3^\circ}$$

$$= \frac{531.2}{138 \times 0.739631}$$

$$\boxed{V = 5.2\ km/h} \quad \text{Ans.}$$

(b) Fractional change in K.E = ?

Now
$$\frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}MV^2 - \left(\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2\right)}{\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2}$$

$$= \frac{MV^2 - m_A v_A^2 - m_B v_B^2}{m_A v_A^2 + m_B v_B^2}$$

$$= \frac{138 \times (5.2)^2 - 83 \times (6.4)^2 - 55 \times (8.8)^2}{83 \times (6.4)^2 + 55 \times (8.8)^2}$$

$$= \frac{3731.52 - 3399.68 - 4259.2}{3399.68 + 4259.52}$$

$$= \frac{-3927.36}{7658.8}$$

$$\frac{K_f - K_i}{K_i} = -0.51$$

$$= -51\ \%$$

Thus ~~of~~ 51% of the initial K.E lost during the collision.

6- Collision in Centre of mass refrence frame.

When collision experiments are performed in laboratory, the measurements are made w.r.t laboratory frame. However the things become easier if the measurements are made w.r.t refrence frame attached to the centre of mass of colliding bodies. Such a frame is called "centre of mass frame".

Consider an elastic collision b/w two particle of masses m_1 & m_2. Suppose, m_2 is at rest before collision and m_1 is moving. So

The position of centre of mass is given by

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{M} \quad \text{——— (1)}$$

where $M = m_1 + m_2$ be the total mass of the system.

The velocity of the centre of mass is given by

$$U_{cm} = \frac{d}{dt}\left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}\right)$$

$$= \frac{m_1\dot{x}_1 + m_2\dot{x}_2}{m_1 + m_2}$$

$$= \frac{m_1U_1 + m_2U_2}{m_1 + m_2}$$

where $U_1 = \dot{x}_1$ = velocity of m_1,
$U_2 = \dot{x}_2$ = Velocity of m_2
in the laboratory frame of refrence.

As m_2 is at rest. So $U_2 = 0$, So the above equation becomes

$$U_{cm} = \left(\frac{m_1}{m_1 + m_2}\right)U_1 \quad \text{——— (2)}$$

Let us now view the collision from centre of mass refrence frame moving w.r.t laboratory frame with velocity V_{cm}

By using Galilean transformation for the two frames S and S'. S is laboratory frame at rest and S' is the centre of mass frame moving w.r.t 'S' with velocity U.

$$V = V' + U$$

Where U = velocity of centre of mass frame S' w.r.t S at rest.
V' = velocity measured in c.m frame S'.
V = " " " laboratory frame S.

We can write

$$V = V' + U_{cm}$$

Let U_1' and U_2' be the initial velocities of the particles m_1 and m_2 w.r.t centre of mass refrence frame. than

$$U_1' = U_1 - U_{cm}$$

By putting the values of U_{cm} from equ. ②

$$\therefore \quad U_1' = U_1 - \left(\frac{m_1}{m_1+m_2}\right)U_1$$

$$U_1' = U_1\left(1 - \frac{m_1}{m_1+m_2}\right)$$

$$= U_1\left(\frac{\cancel{m_1}+m_2-\cancel{m_1}}{m_1+m_2}\right)$$

$$U_1' = \left(\frac{m_2}{m_1+m_2}\right)U_1 \quad \text{———— ③}$$

Similarly $U_2' = U_2 - U_{cm}$

$$U_2' = -U_{cm} \qquad \because u_2 = 0$$

$$U_2' = \left(\frac{-m_1}{m_1+m_2}\right)U_1 \qquad \text{from ② } U_{cm} = \left(\frac{m_1}{m_1+m_2}\right)U_1$$

The final velocities of m_1 and m_2 with m_2 at rest w.r.t laboratory frame of refrence after collision are given by;

$$\left.\begin{aligned} V_1 &= \left(\frac{m_1-m_2}{m_1+m_2}\right)U_1 \\ V_2 &= \left(\frac{2m_1}{m_1+m_2}\right)U_1 \end{aligned}\right\} \quad \text{———— ⑤}$$

The final velocities of m_1 and m_2 with m_2 at rest w.r.t. centre of mass frame of refrence after collision are given by;

$$V_1' = V_1 - U_{cm}$$

Putting v_1 from ⑤ and U_{cm} from ②

$$V_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right)U_1 - \left(\frac{m_1}{m_1+m_2}\right)U_1$$

$$V_1' = \left(\frac{\cancel{m_1}-m_2-\cancel{m_1}}{m_1+m_2}\right)U_1$$

$$V_1' = \left(\frac{-m_2}{m_1+m_2}\right)U_1 \quad \text{———— ⑥}$$

Similarly

$$V_2' = V_2 - U_{cm}$$

$$V_2' = \left(\frac{2m_1}{m_1+m_2}\right)U_1 - \left(\frac{m_1}{m_1+m_2}\right)U_1$$

Putting V_2 from (5) & U_{cm} from (2)

$$V_2' = \left(\frac{2m_1 - m_1}{m_1+m_2}\right)U_1$$

$$V_2' = \left(\frac{m_1}{m_1+m_2}\right)U_1 \qquad \text{———} \quad (7)$$

From equations (3), (4), and (6) and (7) we find that initial velocities of m_1 & m_2 w.r.t. c.m refrence frame are opposite to the final velocities of m_1 and m_2 w.r.t. c.m refrence frame.

Now

K.E is conserved in both laboratory and centre of mass frames of refrence as seen below;

(i) In the laboratory frame, we have;

$$\frac{1}{2}m_1U_1^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

Putting V_1 and V_2 from equation (5), we get

$$\frac{1}{2}m_1U_1^2 = \frac{1}{2}m_1\left[\left(\frac{m_1-m_2}{m_1+m_2}\right)U_1\right]^2 + \frac{1}{2}m_2\left[\left(\frac{2m_1}{m_1+m_2}\right)U_1\right]^2$$

$$\frac{1}{2}m_1U_1^2 = \frac{1}{2}U_1^2\left[\frac{m_1(m_1-m_2)^2 + m_2(4m_1^2)}{(m_1+m_2)^2}\right]$$

$$= \frac{1}{2}U_1^2\left[\frac{m_1(m_1^2+m_2^2-2m_1m_2) + 4m_1^2m_2}{(m_1+m_2)^2}\right]$$

$$= \frac{1}{2}U_1^2\left[\frac{m_1^3 + m_1m_2^2 - 2m_1^2m_2 + 4m_1^2m_2}{(m_1+m_2)^2}\right]$$

$$\frac{1}{2} m_1 U_1^2 = \frac{1}{2} U_1^2 \left[\frac{m_1^3 + m_1 m_2^2 + 2 m_1^2 m_2}{(m_1+m_2)^2} \right]$$

$$\frac{1}{2} m_1 U_1^2 = \frac{1}{2} m_1 U_1^2 \left[\frac{m_1^2 + m_2^2 + 2 m_1 m_2}{(m_1+m_2)^2} \right]$$

$$\frac{1}{2} m_1 U_1^2 = \frac{1}{2} m_1 U_1^2 \left[\frac{\cancel{(m_1+m_2)^2}}{\cancel{(m_1+m_2)^2}} \right]$$

$$\therefore \quad \frac{1}{2} m_1 U_1^2 = \frac{1}{2} m_1 U_1^2$$

So K.E is conserved in laboratory frame of refrence;

(ii) In centre of mass frame of refrence;

Initial K.E before collision $= \frac{1}{2} m_1 U_1'^2 + \frac{1}{2} m_2 U_2'^2$

Putting U_1', U_2' from 3 and 4, we get

$$\text{Initial K.E} = \frac{1}{2} m_1 \left[\left(\frac{m_2 U_1}{m_1+m_2} \right)^2 \right] + \frac{1}{2} m_2 \left[\left(\frac{-m_1 U_1}{m_1+m_2} \right)^2 \right]$$

$$= \frac{1}{2} \frac{U_1^2}{(m_1+m_2)^2} \left[m_1 (m_2^2) + m_2 (-m_1)^2 \right]$$

$$= \frac{1}{2} \frac{U_1^2}{(m_1+m_2)^2} \left[m_1 m_2^2 + m_1^2 m_2 \right]$$

$$= \frac{1}{2} \frac{U_1^2}{(m_1+m_2)^{\cancel{2}}} m_1 m_2 (\cancel{m_2+m_1})$$

$$= \frac{1}{2} \frac{m_1 m_2 U_1^2}{(m_1+m_2)}$$

$$= \frac{1}{2} \frac{m_1 m_2 U_1^2}{(m_1+m_2)}$$

$$= \frac{1}{2} \left(\frac{m_1 m_2}{m_1+m_2} \right) U_1^2$$

$\therefore$ Initial K.E in cm frame $= \frac{1}{2} \mu U_1^2$ ——— (8)

where $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

$$\frac{1}{\mu} = \frac{m_1 + m_2}{m_1 m_2}$$

Now final K.E in the centre of mass frame of refrence is

$$= \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2$$

Putting V_1' & V_2' from (6) and (7), we get

$$\text{Final K.E in c.m frame} = \frac{1}{2} m_1 \left[\left(\frac{-m_2 U_1}{m_1+m_2}\right)^2\right] + \frac{1}{2} m_2 \left[\left(\frac{m_1 U_1}{m_1+m_2}\right)^2\right]$$

$$= \frac{1}{2} \frac{U_1^2}{(m_1+m_2)^2} \left[m_1 (-m_2)^2 + m_2 (m_1)^2\right]$$

$$= \frac{1}{2} \frac{U_1^2}{m_1+m_2} \left[m_1 m_2^2 + m_2 m_1^2\right]$$

$$= \frac{1}{2} \frac{U_1^2}{(m_1+m_2)^2} \left[m_1 m_2^2 + m_2 m_1^2\right]$$

$$= \frac{1}{2} \frac{U_1^2 m_1 m_2}{(m_1+m_2)\cancel{^2}} \left[\cancel{m_2 + m_1}\right]$$

$$= \frac{1}{2} \left(\frac{m_1 m_2}{m_1+m_2}\right) U_1^2$$

$$\boxed{\text{Final K.E in c.m frame is given} = \frac{1}{2} \mu U_1^2} \qquad \text{— (9)}$$

From (8) and (9), we see that

Initial K.E = Final K.E

So K.E is conserved in cm frame of refrence

Inelastic Collision in centre of mass frame of refrence.

In case of completely inelastic collision the mass m_1 on collision with m_2 at rest, lodges itself