

NUCLEAR PHYSICS

54.1 Rutherford Experiment for the Discovery of the Nucleus

In 1897 J. J. Thompson discovered electrons. He also suggested a model of atom. According to his model, an atom consisted equal amount of positive and negative charge. The positive charge of the atom was considered to be spread out through the entire volume of the atom and electrons were thought to be distributed throughout this volume like seeds embedded in a watermelon. Thompson's model of atom could not account for the deflection of the α – particles passing near the surface of the atom and particularly by the backward deflection of an α – particle.

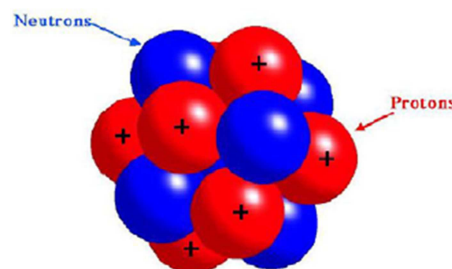
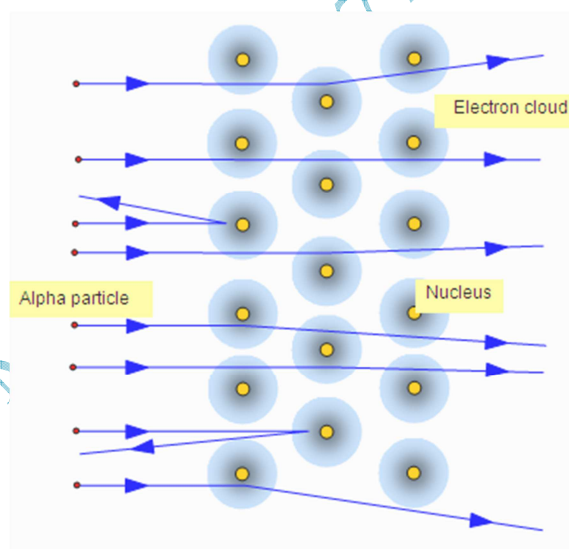
In 1911, Ernest Rutherford performed an experiment to investigate the model of the atom. The apparatus consisted of a source of α – particles, thin gold foil and a moveable detector. A fine beam of high energy α – particles was made to fall on a thin gold foil.

The scattered α – particles were detected at different angle by a moveable detector. It was discovered that most of the α – particles passed through the gold foil without any deflection, or at very small angles of deflection. A very small fraction of α – particles were scattered through large angles approaching 180° .

A graph is plotted between the number of α – particles scattered and the scattering angle as shown in the figure.

The results obtained were very surprising to Rutherford. He concluded that there is a very small region inside an atom which is massive and whole mass of the atom is concentrated at this region which has positive charge. This small region was give the name 'nucleus'. This is the reason why most of the α – particles passed through the gold foil at small scattering angles. Only those α – particles could be deflected through large angles which suffered head-on collision with the nuclei.

Therefore, according to Bohr and Rutherford, the model of atom may consist of a nucleus which is massive and positive charged part, while the electrons are moving around the nucleus in allowed circular orbits.



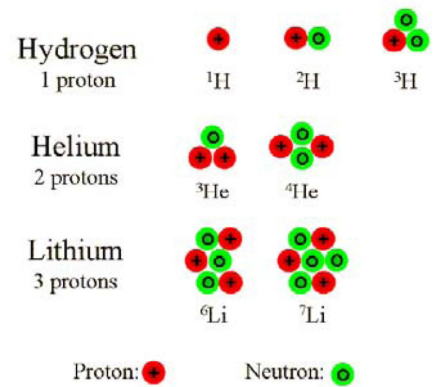
54.2 Some Nuclear Properties

i) Nuclear Systematic

Nuclei are made up of protons and neutrons. But these particles are not true elementary particles; these are made up of other particles called quarks.

The number of protons in a nucleus is called the atomic number or charge number. It is represented by Z . The number of neutrons is called the neutron number. It is represented by N . The total number of nucleon ($N + Z$) is called the mass number. It is represented by A .

The charge on one proton is $+e = 1.6 \times 10^{-19} C$, while the neutron is a neutral particle. The nuclei, having same charge number Z but different mass number is called isotopes. The nuclei which are not stable, used to emit α , β and γ rays are called radioactive nuclides.



54.3 The Nuclear Force

A nucleus is packed with protons and neutrons. As proton is a positively charged particle, so, there must be electrostatic repulsion. But the nucleus is very much rigid and stable. So, there must be some other force inside the nucleus which is responsible for the stability and rigidity of the nucleus. This force is called Strong Nuclear Force.

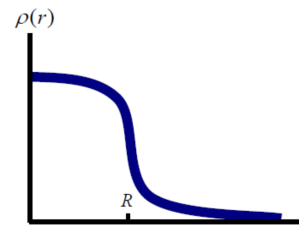
The electrostatic force (Coulomb force) is a long range force, but the Strong Nuclear Force is a short range force, having the range of $10^{-15} m$. This force binds every nuclear pair: proton-proton, neutron-neutron and proton-neutron pair within the tiny nuclear volume. For the short range of the order of $10^{-15} m$, the coulomb force is much smaller than the Strong Nuclear Force. If the separation between the nucleons is increased beyond $10^{-15} m$, the strong nuclear force drops to zero rapidly, then the Coulomb's force of repulsion will be able to break the nucleus.

54.4 Nuclear Radii

The Bohr radius is $a_0 = 5.29 \times 10^{-11} m$, while the radius of the nucleus is of the order of $10^{-15} m$. So, the nuclei are smaller than the atoms by a factor of 10^4 . The size and structure of the nuclei can be studied by scattering experiments; using incident beam of high energy electrons. The energy of incident electrons must be greater than 200 MeV. These experiments measure the diffraction pattern of the scattered particles and deduce the shape of the scattering object (the nucleus).

The nucleus does not have sharply defined surface, however have a characteristic radius ' R '. The density ρ_r has constant value inside the nucleus, but it falls to zero through fuzzy surface zone. The mean radius is given by:

$$R = R_0 A^{1/3}$$



Where A is the mass number and R_0 is a constant whose value is 1.2 fm. For example, the ^{63}Cu has the radius $R = (1.2 \text{ fm}) (63)^{1/3} = 4.3 \text{ fm}$.

54.5 Nuclear Mass and Binding Energies

Atomic masses are measured in atomic mass units (μ). The atomic mass unit is defined as $\frac{1}{12}$ th times the atomic mass of ^{12}C . Thus

$$1 \mu = 1.6605 \times 10^{-27} \text{ kg}$$

One atomic mass unit ' 1μ ' is equivalent to the energy:

$$E = \Delta m c^2 = (1.6605 \times 10^{-27} \text{ kg})(3 \times 10^8 \text{ m s}^{-1})^2$$

$$E = 1.49 \times 10^{-10} \text{ J}$$

$$E = \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E = 0.931 \times 10^9 \text{ eV}$$

$$E = 931 \text{ MeV}$$

This means that we can write c^2 as 931 MeV/u .

54.6 Binding Energy

It is the amount of energy required to tear a nucleus into its constituent nucleons. Or when nucleons are fused to form a nucleus, then some energy is released which is called binding energy.

Example: The nucleus of Deuteron (a heavy hydrogen atom) consists of a protons and a neutron bounded together by a strong nuclear force. The energy E_B that we must add to deuteron to tear it apart in to its constituent nucleons is called the energy. If m_d , m_n and m_p are the masses of deuteron, neutron and proton respectively, then according to the law of conservation of energy:

$$m_d c^2 + E_B = m_n c^2 + m_p c^2$$

$$(m_d c^2 + m_e c^2) + E_B = m_n c^2 + (m_p c^2 + m_e c^2)$$

$$m({}_1\text{H}^2) c^2 + E_B = m_n c^2 + m({}_1\text{H}^1) c^2$$

$$E_B = m_n c^2 + m({}_1\text{H}^1) c^2 - m({}_1\text{H}^2) c^2 \quad \text{----- (1)}$$

Here $m({}_1\text{H}^1)$ and $m({}_1\text{H}^2)$ are the masses of the hydrogen and deuteron atom, respectively.

$$E_B = [m_n + m({}_1\text{H}^1) - m({}_1\text{H}^2)] c^2 = \Delta m c^2 \quad \text{----- (2)}$$

In which, $\Delta m = m_n + m({}_1\text{H}^1) - m({}_1\text{H}^2)$.

As $m_n = 1.008665 \text{ u}$, $m({}_1\text{H}^1) = 1.007825 \text{ u}$ and $m({}_1\text{H}^2) = 2.014102 \text{ u}$. So by substituting the values (2) and replacing c^2 by its equivalent 931 MeV/u , we find the binding energy to be:

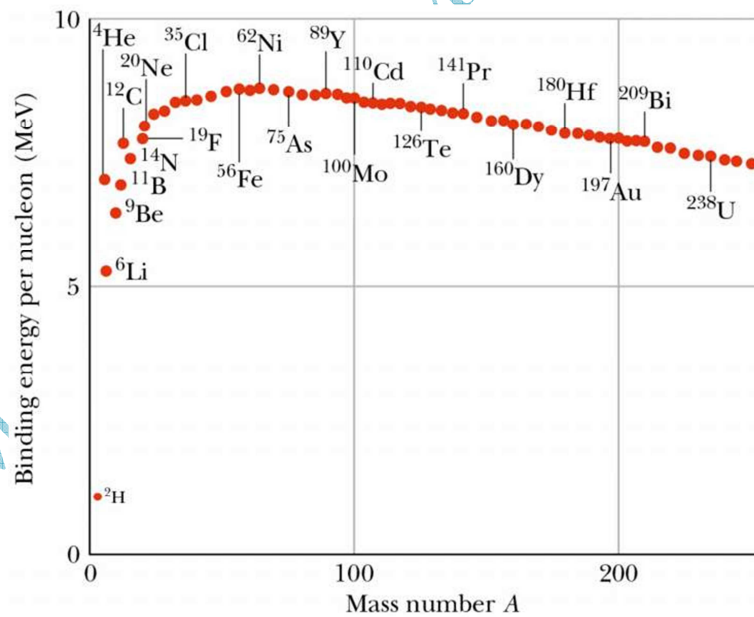
$$E_B = 2.224 \text{ MeV}.$$

54.7 Binding Energy per Nucleons

This is the binding energy of the deuteron. If the binding energy E_B is divided by mass number A , then binding energy per nucleon $\left(\frac{E_B}{A}\right)$ is obtained. Figure show a graph between energy per nucleon $\left(\frac{E_B}{A}\right)$ and mass number A . The $\left(\frac{E_B}{A}\right)$ is high for middle mass nucleons. It means that these nucleons are tightly bound and the nucleus is much stable. The region of greatest stability corresponds to mass number about 50 to 80.

The binding energy per nucleon curves drops at both high and low mass numbers has the practical consequence of the greatest importance. The dropping of the binding energy curve at high mass numbers tells us energy can be released in the nuclear fission of a single massive nucleus into two smaller fragments.

The dropping of the binding energy curve at low mass numbers tells us that energy will be released if the two nuclei of small mass numbers combined to form a single middle mass nucleus. This process, the reverse of fission, is called nuclear fusion.



54.8 Nuclear Spin and Magnetism

Just like atoms, the nuclei have intrinsic angular momentum (nuclear spin) whose maximum value along z-axis is $J\hbar = J \frac{h}{2\pi}$, where J is total intrinsic angular momentum or nuclear spin quantum number.

In atomic magnetism, the Bohr magneton μ_B is given by:

$$\mu_B = \frac{eh}{4\pi m_e} = 5.79 \times 10^{-5} \frac{eV}{T}$$

Where m_e is the mass of electron and T stand for tesla. Similarly for nuclear magneton μ_N , we have:

$$\mu_N = \frac{eh}{4\pi m_p} = 3.15 \times 10^{-8} \frac{eV}{T}$$

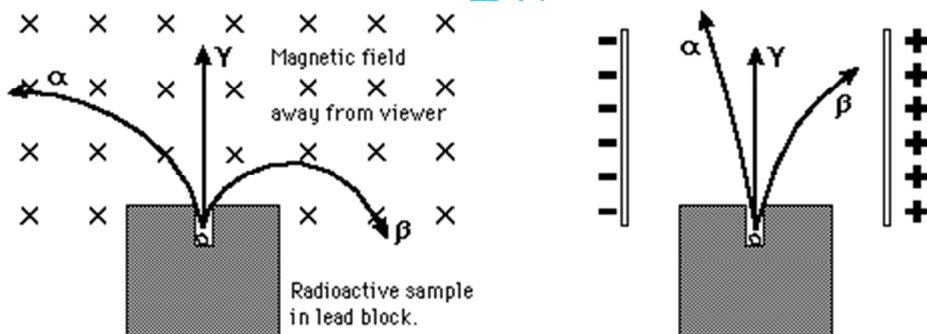
The magnetic moment of the heavier nuclei can be analyzed in terms of the magnetic moments of its constituent protons and neutrons.

54.9 Radioactive Decay

The elements having charge number greater than 82, are not stable. They disintegrate and emit particles or radiation spontaneously. Such elements are radioactive and this process of spontaneous emission of particles or radiations is called radioactive decay or natural radioactivity.

Experiment.

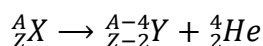
The radioactive sample is placed in a cavity in a lead block above which a photographic plate is held. The whole apparatus is placed inside a vacuum chamber, in which magnetic field is also applied, as shown in the figure.



Three images are obtained on the photographic plate. It means that three types of radiations are emitted by the radioactive sample, which α –particles, β –particles and γ –rays.

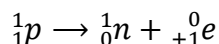
α –particles

α –particles is a helium nucleus which consist of two protons and two neutrons. Its charge number is 2 and mass number is 4. It is highly ionizing particle, but its range is small. Emissions of α –particles from an atom reduces its mass number A by four units and atomic number Z by two units. As a result, the product element moves two places backward in the periodic table.

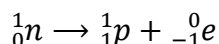


β –particles

A β –particle is a positive or negative electrons which is emitted from the nucleus of a radioactive element. When a proton in the nucleus transforms into a neutron, a positive β –particles is emitted.



When a neutron inside a nucleus changes into protons, then negative β –particles are emitted:



Emission of negative β –particles leaves the mass number of the product nucleus unchanged, but the atomic number (charge number) increases by one unit. So the product element moves one place ahead in the periodic table.

Similarly, emission of positive β –particles leaves the mass number of the product nucleus unchanged but the atomic number (charge number) decreases by one unit. So the product element moves one place backward in the periodic table.

 γ –Rays

γ –rays are not material particles but they are electromagnetic rays moving with the velocity of light. These are the most energetic electromagnetic radiations having shortest wavelength and largest frequency. γ –ray can produce photoelectric effects, Compton effect and pair production. These radiations have largest range. γ –rays are emitted from the excited nuclei of radioactive elements.

Emission of γ –rays does not change the mass number and atomic number of the product nucleus.

**54.10 Laws of Radioactivity****54.10.1 Half Life**

It is the time during which one half of the atoms of the parent element decay in to daughter element. The half life of a radioactive element may vary from fraction of a second to millions of year.

The radioactive decay obeys the following two statistical laws:

- The number of atoms that decay at any instant is proportional to the number of atoms present at that instant.
- No sample of radioactive element can ever completely decay in a finite time.

Let N_0 is the number of atoms initially present at time $t = 0$. Let N is the number of atoms at any time t . If dN are the number of atoms decay during the interval of time dt , then the rate of decay $-\frac{dN}{dt}$ is directly proportional to the number of atoms present. i.e.,

$$-\frac{dN}{dt} \propto N$$

$$-\frac{dN}{dt} = \lambda N$$

Where λ is the decay constant. It is the characteristics of radioactive element.

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^N \frac{dN}{N} = -\int_0^t \lambda dt$$

$$|\ln N|_{N_0}^N = -\lambda |t|_0^t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t} \quad \text{----- (1)}$$

This is known as radioactive decay law.

The rate of decay $\left(-\frac{dN}{dt}\right)$ i.e., activity R can be find out by differentiating equation (1):

$$\frac{dN}{dt} = N_0 \frac{d e^{-\lambda t}}{dt}$$

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t} \quad \text{----- (2)}$$

Where $R_0 = \lambda N_0 =$ Decay rate at $t = 0$.

It is clear from equation (1) and (2), that radioactive follows an exponential law and it takes an infinite time to decay a radioactive element completely.

When $N = \frac{N_0}{2}$, then $t = T_{1/2} = \text{Half life}$

Putting values in equation (1), we have:

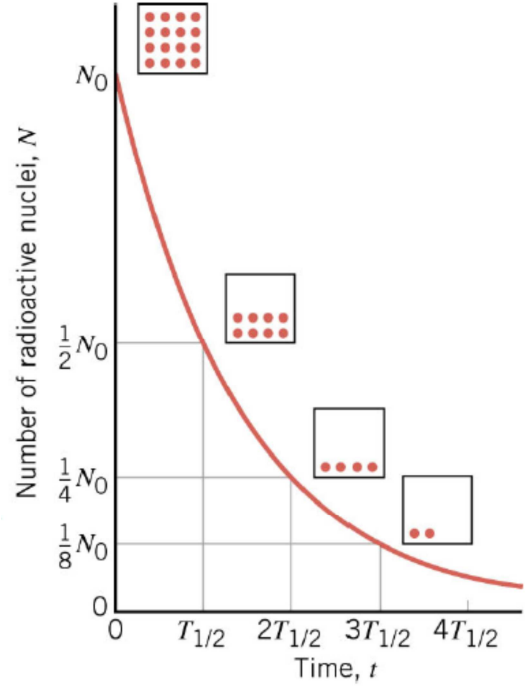
$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-(\lambda T_{1/2})}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda T_{1/2}$$

$$\ln 2 = \lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$



This is the expression of half life of a radioactive element.

54.10.2 Mean Life

As the possible life of a radioactive element varies from 0 to ∞ , so the total life of all N_0 initially present atoms in a given sample can be find out by the expression $\int_0^\infty t dN$.

The mean or average life T^* for a sample can be described by the expression:

$$T^* = \frac{\text{Total Life Time}}{\text{Total Number of Atoms}} = \frac{\int_0^\infty t dN}{\int_{N_0}^0 dN}$$

$$\int_{N_0}^0 dN = [N]_{N_0}^0 = 0 - N_0 = -N_0$$

Also,

$$\frac{dN}{dt} = -\lambda N$$

$$dN = -\lambda N dt$$

$$dN = -\lambda N_0 e^{-\lambda t} dt$$

$$\text{as } N = N_0 e^{-\lambda t}$$

$$T^* = \frac{\int_0^\infty t (-\lambda N_0 e^{-\lambda t} dt)}{-N_0}$$

$$T^* = \int_0^\infty t \lambda e^{-\lambda t} dt$$

$$T^* = \lambda \int_0^\infty t e^{-\lambda t} dt$$

Integrating by parts, we get:

$$T^* = \lambda \left[\left| \frac{te^{-\lambda t}}{-\lambda} \right|_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$T^* = \lambda \left[0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right] = \lambda \left[\frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right]$$

$$T^* = \int_0^\infty e^{-\lambda t} dt$$

$$T^* = \left| \frac{e^{-\lambda t}}{-\lambda} \right|_0^\infty = -\frac{1}{\lambda} [e^{-\infty} - e^0] = -\frac{1}{\lambda} \left[\frac{1}{e^\infty} - 1 \right] = -\frac{1}{\lambda} [-1]$$

$$T^* = \frac{1}{\lambda}$$

This is the expression of mean life of a radioactive element.

As

$$T_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \times \frac{1}{\lambda}$$

$$T_{1/2} = 0.693 \times T^*$$

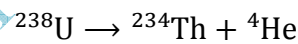
This expression shows the mean life of a radioactive element is greater than its half life.

$$T^* = \lambda \left[t(0) + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right]$$

$$T^* = \lambda \left[\frac{1}{\lambda} \left| \frac{e^{-\lambda t}}{-\lambda} \right|_0^\infty \right]$$

54.11 Alpha Decay

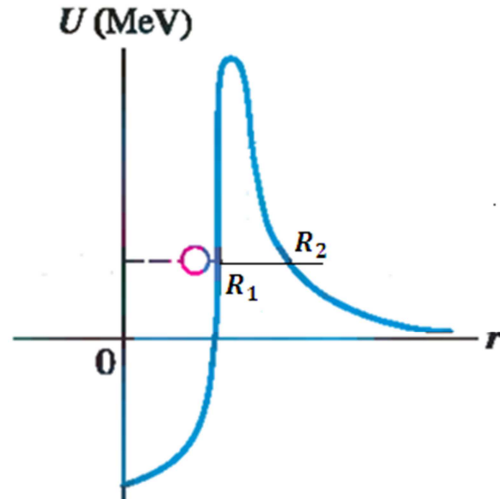
The radionuclide ^{238}U decays spontaneously according to the scheme:



With the half life of 4.47×10^9 years. In this process, an energy of 4.27 MeV is emitted appearing as the kinetic energy of the alpha particle (^4He) and the recoiling residual nucleus (^{234}Th).

In order to explain the alpha decay, a model is used in which the α – particle is assumed to be exist preformed inside the nucleus before it escapes.

The figure shows the approximate potential energy function $U(r)$ for the α – particle and the residual ^{234}Th nucleus as the function of their separation. It is a combination of a potential well associated with the attractive strong nuclear force that acts in the nuclear interior ($r < R_1$) and a Coulomb's potential associated with the repulsive electrostatic force that acts between the two particles after the decay has occurred ($r > R_2$).



The line that intersect the potential energy curve at point R_1 and R_2 , is the measure of emitted during one alpha decay. The energy of 4.27 MeV is emitted appearing as the kinetic energy of the alpha particle (^4He) and the recoiling residual nucleus (^{234}Th).

The decay of the alpha particle is accompanied by the emission of energy. Now the question arises that why did ^{238}U not decay shortly after they were created?

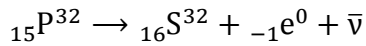
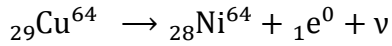
The answer to this question is given is background of the potential barrier consideration. We can visualize this barrier as a spherical shell whose inner radius is R_1 and whose outer radius is R_2 , its volume being forbidden to the α – particle under the law of classical physics.

But according to the quantum mechanics, there is a chance of tunneling through barrier. It is this tunneling due to which an alpha particle gets a chance to come out of nucleus. But its probability is extremely little, (1 out of 10^{38}). It is one about in 10^9 years. That is why ^{238}U nucleus has such long half life.

If energy of alpha particle is comparatively high, the potential barrier will be appear to it thinner and lower. Hence in this case, tunneling will occur more readily, and half life is reduced considerably. For example, alpha particle emitted by ^{228}U has energy of 6.81 MeV.

54.12 Beta Decay

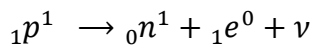
Emission of electron or a positron from a nucleus is known as beta decay. Beta particles don't exist in nucleus. They are emitted as soon as they are formed by disintegration of a neutron or a proton in a nucleus. The examples of beta decay are as follows:



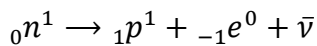
Here ${}_1\text{e}^0$ and ${}_{-1}\text{e}^0$ are electron and positron, while ν and $\bar{\nu}$ are neutrino and anti-neutrino respectively.

Beta decay is accompanied by emission of another particle known as neutrino or anti-neutrino. Neutrino is emitted with positron (β^+) and anti-neutrino is emitted with electron (β^-).

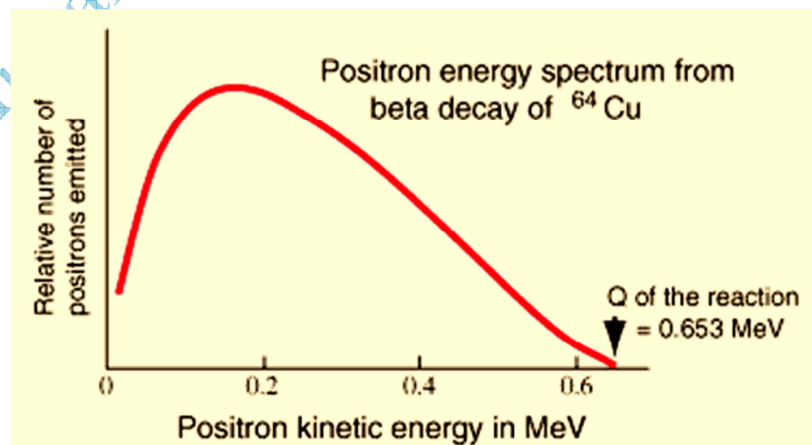
In positive beta decay, a proton disintegrates into a neutron and a proton along with the emission of a neutrino.



In negative beta decay, a neutron disintegrates into a proton and an electron along with the emission of anti-neutrino.



The kinetic energy of beta particle does not remain constant, but it changes over a range as shown in the figure.



The maximum value of kinetic energy of the beta particle is 0.653 MeV, which is also the total disintegration energy for the case of beta decay from Cu. The kinetic energy of the

beta particle changes over the range because the disintegration energy is shared by beta particle and neutrino.

Beta particle is not emitted alone. But with every beta particle another particle is emitted, called neutrino. Total energy of beta particle and neutrino is quantized. But the disintegration energy is shared by the beta particle and neutrino in any proportion, i.e., beta particle may take any energy between 0 and maximum. The rest of energy is carried by neutrino.

The neutrino and anti-neutrino are very light particles having no charge and mass nearly of the order of $\frac{1}{200}$ of mass of electron. So they interact with matter weakly and are very difficult to detect.

54.13 Units for Measuring Ionizing Radiation

Becquerel.

SI unit of radioactivity is Becquerel. One Becquerel is one disintegration per second.

The Curie.

The unit of activity or rate of decay of a radioactive source is Curie Ci. It is defined as the activity of one gram of radium in equilibrium.

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegrations/second}$$

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Becquerel}$$

The Roentgen

It is the unit of exposure which may be defined as the exposure of beam of x – rays or γ – rays to produce 1.6×10^{12} ion pair per gram of air, the air being dry and at standard temperature and pressure.

The Rad

This is acronym for radiation absorbed dose and is a measure of the dose actually delivered to a specified object. The specified part of a body (the hand, say) is said to have received and absorbed dose of 1 rad when 10^{-5} J/g have been delivered to it by ionizing radiation.

The rem

This is acronym for roentgen equivalent in man and is a measure of dose equivalent. The dose equivalent (in rem) is found by multiplying the absorbed dose (in rad) by quality factor QF. According to the recommendation of the National Council on Radiation

Protection, no individual who is exposed to radiations should receive a dose equivalent greater than 500 m rem (0.5 rem) in any one year.

54.14 Radioactive Dating

The age of a sample can be determined by radioactive dating. Suppose an initial radio nuclide I decays to a final product F with a known half-life $T_{1/2}$.

At particular time $t = 0$, we start with N_0 initial nuclei with product (final) nuclei equal to zero. After a time t , the initial nuclei N_0 at $t = 0$ reduce to ' N_I ' with the product (final) nuclei ' N_f ', where $N_f = N_0 - N_I$.

The initial nuclei decay according to the exponential law:

$$N_I = N_0 e^{-\lambda t}$$

$$\frac{N_I}{N_0} = e^{-\lambda t}$$

Taking logarithm on both sides, to the base e, we get:

$$\ln\left(\frac{N_I}{N_0}\right) = -\lambda t$$

$$t = \frac{-1}{\lambda} \ln\left(\frac{N_I}{N_0}\right) = \frac{1}{\lambda} \ln\left(\frac{N_0}{N_I}\right)^{-1}$$

$$t = \frac{1}{\lambda} \ln\left(\frac{N_0}{N_I}\right)$$

$$\text{As } T_{1/2} = \frac{\ln 2}{\lambda} \text{ or } \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$$

So,

$$t = \frac{T_{1/2}}{\ln 2} \ln\left(\frac{N_0}{N_I}\right)$$

$$\text{Put } N_0 = N_I + N_f$$

$$t = \frac{T_{1/2}}{\ln 2} \ln\left(\frac{N_I + N_f}{N_I}\right)$$

$$t = \frac{T_{1/2}}{\ln 2} \ln\left(1 + \frac{N_f}{N_I}\right)$$

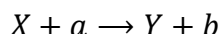
Here t is the age of the original radioactive nuclide (element). It can be measured by the ratio $\frac{N_f}{N_I}$. Here $T_{1/2}$ is the half-life of the radioactive element which is known.

This method can be used to determine the time since the formation of the solar system. Example include the ration of U^{238} to Pb^{206} , Rb^{87} to Sr^{87} and K^{40} to Ar^{40} .

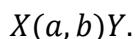
The terrestrial rocks, moon rocks are analyzed by these method, all seem to have common age of around 4.5×10^9 year, which we take to be the age of our solar system.

54.15 Q –Energy in Nuclear Reactions

A nuclear reaction can be represented by:



Or in more compact notation



Here X is the target nucleus, a is the projectile nucleus, Y is the residual nucleus and b is the emerging nucleus. The projectile particle a may be charged particle which can be accelerated Van de Graph accelerator or cyclone, or it may be a neutron from the nuclear reactor.

When the projectile particle penetrates a target nucleus, then a nuclear reaction takes place. The reaction energy Q is defined as (rest mass energies)

$$Q = (m_X + m_a)c^2 - (m_Y + m_b)c^2 \quad \text{-----} \quad (1)$$

Here $m_X = \text{mass of the target nucleus}$

$m_a = \text{mass of the projectile nucleus}$

$m_Y = \text{mass of the residual nucleus}$

$m_b = \text{mass of the emerging nucleus}$

If ' K ' represents the kinetic energy, then the reaction energy ' Q ' is given by:

$$Q = (K_Y + K_b) - (K_X + K_a) \quad \text{-----} \quad (2)$$

Equation (1) and (2) are only valid, when Y and b are in their ground state.

Exothermic

If the reaction energy has positive value i.e., $Q > 0$, then such nuclear reaction is known as exothermic .

Endothermic

If the reaction energy Q is negative i.e., $Q < 0$, then such a nuclear reaction is known as endothermic. Such a reaction will not “go” unless a certain minimum kinetic energy (the threshold energy) is carried into the system by the projectile. It means that endothermic reaction needs some certain energy for its performance.

Scattering

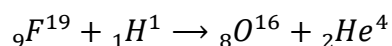
A nuclear reaction is called scattering, if the particles ' a ' and ' b ' are identical and so ' X ' and ' Y ' are also identical.

Elastic Scattering

If the kinetic energy of the system before the reaction is equal to the kinetic energy after collision ($Q = 0$), and all nuclei remains in their ground state, then such scattering is known as elastic scattering.

Inelastic Scattering

If the kinetic energies of the system are different before and after the reaction i.e., $Q \neq 0$, then it is called inelastic scattering. In this case 'Y' and 'b' may be left in excited states.

Example

For this reaction $Q=8.13$ MeV. In this reaction, the system loses rest mass energy and gives some kinetic energy.



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